TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 509

TANK TESTS OF FLAT AND V-BOTTOM PLANING SURFACES

By James M. Shoemaker
Langley Memorial Aeronautical Laboratory

Washington
November 1934

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NACA-TN-509

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SUMMARY

Four planing surfaces, all having beams of 16 inches and lengths of 60 inches but varying in dead rise by 10° increments from 0° to 30°, were tested in the N.A.C.A. tank. The results cover a wide range of speeds, loads, and trim angles, and are applicable to a variety of problems encountered in the design of seaplanes.

The data are analyzed to determine the characteristics of each surface at the trim angle giving minimum resistance for all the speeds and loads tested. A planing coefficient intended to facilitate the application of the results to design work is developed and curves of resistance, wetted length, and center of pressure are plotted against this coefficient. Several examples, showing the application of the test data to specific design problems, are included.

INTRODUCTION

The bottom of a seaplane hull or stepped hydroplane is ordinarily made up of two planing surfaces, one running in the wake of the other, separated by a discontinuity, or step. In the case of the seaplane hull most of the load is carried on the forward surface, or forebody. The function of the afterbody is primarily to provide longitudinal stability at low speeds. Tests of complete hulls give only the combined effect of these two surfaces, including the interference of the forebody wake on the afterbody. A study of simple planing surfaces offers a means of segregating the performance of the working portion of the boat bottom from the effects of other parts that may be necessary in a seaworthy boat, but do not aid in carrying the load.

Previous work on the problem of planing surfaces (references 1 to 5) has furnished a good understanding of the behavior of planing boats. In general, such craft are distinguished from displacement boats by the fact that a part of the lifting force is supplied by the dynamic reaction of the water against the bottom, arising from the downward momentum imparted by the forward motion of the inclined bottom surface. A considerable variety of bottom shapes are capable of providing planing lift, but all such shapes have several characteristics in common. The longitudinal sections approximate straight lines, terminating at a sharp discontinuity at the after end. In a boat with more than one planing surface this discontinuity is known as a "step." In order to furnish an upward dynamic reaction, it is necessary that the water flowing off at the step be directed downward from the free-water surface. Planing bottoms usually intersect the sides of the boat in sharp chines, to prevent the water from following up the vertical surface. Other methods, such as lapped strakes, have been used successfully to accomplish the same purpose.

At rest and at low forward speeds the planing boat is, of course, supported by buoyant forces. As the speed is increased the high pressures resulting from the dynamic reaction on the bottom cause the water to flow out from under the chines and step with sufficient momentum to clear the adjacent vertical surfaces. The transition from floating to planing thus takes place gradually as the speed is increased. For the purposes of this paper, planing will be defined as the condition wherein the vertical surfaces of the hull are running entirely dry; that is, only the bottom, bounded by the chines and step, is in contact with the water. It should be noted that even in this condition, the load is not entirely carried by dynamic lift. The bottom surface of the boat is still below the free-water level; consequently there is a hydrostatic reaction acting on it in addition to the force required to give downward momentum to the water.

The present tests are intended to give numerical data on the resistance, center of pressure, and wetted length of a planing surface running at any load, speed, and trim angle within the useful planing range. The previous experimental work on the subject has covered only a rather small range of speeds and loads, and the theoretical treatment to date does not offer a workable means of extending the results to cover all cases of interest to the designer. The subjects of pressure distribution on a planing

bottom and of wave formation are not treated here. Sottorf's excellent experiments on these phases of the problem (references 2 and 3), although necessarily limited to relatively few speeds and loads, give adequate information on the phenomena involved and offer a sound basis for future theoretical work..

The tests covered in this report were made in the N.A.C.A. tank during November 1933. They include force measurements on four planing surfaces all having a beam of 16 inches but varying in dead rise from 0° to 30° by 10° increments. The results are directly applicable to the design of planing boats having straight V bottoms, and also offer a means of determining the effect of dead rise, unobscured by the influence of the other design characteristics of an actual boat hull. (See reference 6.)

APPARATUS AND PROCEDURE

Apparatus.- The N.A.C.A. tank, in which the present experiments were conducted, is described in detail in reference 7. The apparatus used in the planing-surface tests has been altered somewhat from that described in the reference. The principal change is in the method of suspending the model. The present arrangement is shown schematically in figure 1. It is somewhat more compact than the older form of towing gate, and eliminates the necessity of correcting for interaction of the load, resistance, and trimming moment on the various measurements. The entire gear is suspended by two tapes connected to the counterweight dashpot and rises and falls as a unit, instead of being pivoted at one end. In the previous apparatus the reaction applied to this pivoted end by the pitching moment of the restrained model caused an error in the load on the water that was negligibly small for seaplane hulls, but became serious for the planing surfaces because of the wide range of pitching moments encountered. In the present apparatus this error is not present. The vertical counterweights shown in figure 1 were added to remove the effect of any slight acceleration of the carriage during the test run. The weight of the model and gear is so great compared to the resistance being measured that a consistent acceleration, although so small as to be imperceptible on the speed record, will cause an appreciable error on the resistance record if the system is not balanced against longitudinal accelerations.

The apparatus used to measure the trim angle and moments in these tests is similar to that described in reference 7. The model is restrained by a flat spring controlled by means of an electric motor. The position of the spring is set by the observer to hold the model at the desired trim angle. The deflection of the spring from its free position is calibrated to give the trimming moment imposed on the model.

Models.- The principal dimensions of the planing surfaces used in this investigation are shown in figure 2. The models were made of mahogany, and the bottom surfaces were finished with gray enamel. Transverse stripes spaced 2 inches apart and numbered to correspond to the distance from the trailing edge in inches were painted on the bottoms of the models for the purpose of reading the wetted length. The bottoms of all models consisted of plane surfaces, intersecting in a sharp edge at the keel for the three V-bottom models.

Procedure.- The experiments were conducted in the same manner as that used for "complete" tests of flying-boat hulls at the N.A.C.A. tank. The procedure, described in reference 8, consists of running the model at a series of predetermined speeds, loads, and trim angles, and measuring the resulting resistance, trimming moment, and draft. In the present tests the wetted length, or distance from the trailing edge to the farthest forward point in contact with the water, was also read. In the case of the V-bottom surfaces this point of contact is sharply defined as the intersection of the keel with the free-water surface and was read in a mirror set close to the water surface several feet to one side of the model. The flat surface, however, builds up above the free-water surface a small "roll" of water, which emerges from the sides in the form of spray. The wetted length for the flat surface was read on the side of the surface at the forward portion of this emerging spray. Sottorf's experiments (reference 2) show that the curvature of the intersection of the roll of water with the flat surface is very slight across the beam and that this method of reading the wetted length gives results which are satisfactory for design purposes.

Test schedule.- The schedule of loads, speeds, and trim angles was determined to conform to the range of these variables met in flying-boat practice. At the low speeds and heavy loads corresponding to the floating condition for a boat hull measurements were, of course, im-

possible. The lower limit of speed for a given load was determined in the case of low trim angles by the wetted length approaching the over-all length of the model and, in the case of high trim angles, by the tendency of the wave at the side of the model to fall inward and foul the stern bulkhead. The latter condition corresponds to the lower limit of the planing condition as defined in the introduction to this paper. Higher loads could have been carried at low trim angles if longer surfaces had been used; the length chosen, however, is already somewhat greater in proportion to the beam than that of the longitudinally straight portion of most flying-boat hulls. The upper limit of speed was chosen to correspond to the average get-away speed of flying boats. In the case of the flat surface operating under light loads, an upper limit of the speed was fixed by the fact that the model started chattering on the water surface when the wetted length became very short. The resulting motion was a rapid vibration in pitch of sufficient violence to prevent accurate measurements. The tests at the heavier loads were carried out to high speeds in all cases. This condition is not of interest in a flying-boat hull because the wing lift reduces the load at high speeds, but it does represent the case of a planing surface craft and is of some value in the general study of planing phenomena.

Tare corrections.- The windage tare for these tests was determined with the model attached to the towing gear. The trim angle was set at 0° and the height was adjusted so that the keel cleared the water by 2 inches. Several trim angles and water clearances were tried, and the effect of changes in these quantities was found to be small for trim angles up to 8° and water clearances down to 1 inch. This method of determining the windage tare differs from that used in tests of seaplane-hull models in that the air drag of the model is not included in the water resistance. The water resistance does include, however, the interference between the bottom of the model and the water when the surface is running on the water. This procedure seems to be logical, because a similar interference is present in the case of a planing boat running on the water. No attempt was made to determine the tare effects of air forces on the trimming moment. The moments used in the final computations are those actually measured on the models, corrected only for the moment about the point of suspension produced by their weights.

RESULTS

Test data.- The corrected values of resistance, wetted length (w.l.), center of pressure (c.p.), and draft for each speed, load, and trim angle are given in tables I to IV for the four models. The values given are the direct observations for each test point with the exception of the resistance and the center of pressure. The center of pressure was determined graphically from the known trimming moment and load-resistance ratio as the intersection of the resultant water force with the keel line of the model, and is given as the distance in inches from this intersection to the trailing edge of the bottom.

The data given in the tables, with the exception of the drafts, are plotted in figures 3 to 19. The resistance values read from the curves in these figures were cross-plotted against trim angle to determine the minimum resistance and the best trim angle for each load at a series of selected speeds. The wetted lengths and centers of pressures for the same loads and speeds were then determined for the best trim angles from cross-plots of these quantities against angle. The results of these operations were reduced to nondimensional form, and are presented as curves of resistance coefficient, best trim, angle, wetted length, and center of pressure, all plotted against speed coefficient, in figures 20 to 27. The coefficients are the same as those used in the analysis of the test results of flying-boat hulls. (See reference 8.) They conform to Froude's law of similitude, and are defined as follows:

Load coefficient, $C_\Delta = \frac{\Delta}{wb^3}$

Resistance coefficient, $C_R = \frac{R}{wb^3}$

Speed coefficient, $C_V = \frac{V}{\sqrt{gb}}$

where

Δ is the load on the water, lb.

R, resistance, lb.

w, weight density of water, lb./cu.ft. (63.5 lb./cu.ft. for water in N.A.C.A. tank)

b, beam, ft.

V, speed, ft./sec.

g, acceleration of gravity, ft./sec.^{2}

The wetted length and center-of-pressure distance in inches have been divided by the beam in inches to reduce these quantities to nondimensional form. The results given in figures 20 to 27 may thus be used with any other consistent system of units.

Precision.- The apparatus used in the present tests gives measurements that are correct within the following limits:

Load on water	±0.3 lb.
Resistance	±.1 lb.
Trimming moment	±1.0 lb.-ft.
Draft	±.1 in.
Wetted length	±1.0 in.

Unsteady running of the models causes the scattering of the test points to be somewhat greater than these variations in particular cases. The accuracy of the values of the center-of-pressure location depends upon that of the trimming-moment readings. For a load of 80 pounds, an error of 1 pound-foot causes an error of 1/80 foot, or 0.15 inch, in the center-of-pressure location. The same error in trimming moment, however, causes an error of 1/5 foot, or 2.4 inches, for a load of 5 pounds. The scattering of the values of center-of-pressure location is consequently large for the lighter loads, and the curves of this quantity for loads less than 20 pounds have been omitted in some cases.

DISCUSSION

Variations with speed.- The trend of the curves of resistance, wetted length, and center of pressure against speed may be seen in figures 3 to 27. For low angles of dead rise, the resistance for a given load and trim angle tends, in general, to decrease with increasing speed. With increased dead rise this tendency disappears and the curves for model 30, having the highest dead-rise angle tested, show a pronounced increase in resistance with increasing speed. The wetted lengths and distances from the trailing edge to the centers of pressure decrease consistently with increasing speed for all loads, trim angles, and angles of dead rise. The trim angle τ_0 giving minimum resistance varies only slightly with speed for all the plates except model 30, having 30° dead rise. For that model, τ_0 increases with speed through a range of about 2° when the speed coefficient is increased from 2.0 to 5.0. The tendency of τ_0 to remain nearly constant with speed is contrary to the results obtained from flying-boat tests. The apparent discrepancy may be ascribed in part to the effect of the afterbody. The greater portion of the difference, however, probably arises from the fact that the bluff bow of the boat hull runs in the water at low speeds and low trim angles, causing high resistance. Consequently, for speed coefficients between 2.0 and 3.0 increasing the trim angle of the hull reduces the resistance by lifting the bow out of the water. The planing surfaces within the limits of their over-all length are not subject to this cause of high resistance at low trim angles, and consequently the optimum value of the trim angle is somewhat lower than that for a boat hull. At higher speeds, where the planing bottom of the boat hull is long enough to keep the longitudinally curved portion clear, the values of the best trim angle for the hulls and planing surfaces agree fairly well.

Variations with dead rise.- For a given speed, load, and trim angle, an increase in the dead-rise angle generally causes an increase in the resistance, wetted length, and distance from the trailing edge to the center of pressure. There is also a definite increase of the trim angle giving minimum resistance with increasing dead rise, as may be seen in figures 20 to 27. The effect of dead-rise angle on the load-resistance ratio is of particular interest in seaplane design, because of the necessity of keeping the landing shock as low as is consistent with

good planing characteristics. Figure 28 shows the effect of dead-rise angle on the load-resistance ratio for a number of values of the speed and load coefficients. It will be seen that, in general, the smaller dead-rise angles give higher values of Δ/R. This effect is particularly pronounced at high speeds.

Development of planing coefficient.- In order to facilitate the use of the test results in design work, and to provide a means of extrapolating to higher speeds and loads than those tested, the values of load-resistance ratio, wetted length divided by beam, and center-of-pressure distance divided by beam are plotted against a planing coefficient

$$K = \frac{\Delta}{\frac{1}{2}\,\rho\,V^2\,b^2}$$

in figures 29 to 32 for the four planing plates. The coefficient K is based upon the formulas developed by Schröder in references 9 and 10. The assumptions involved in Schröder's formulas are that in the planing condition the effect of gravity forces on the flow may be neglected and that the friction coefficient does not change with speed. All of the load is thus assumed to be carried by the dynamic reaction of the water; hence if the ratio of Δ/V^2 is held constant for a given plate form, size, and trim angle, the values of Δ/R, w.l./b, and c.p./b will also remain constant. The coefficient is made nondimensional by dividing Δ/V^2 by $\rho b^2/2$ where ρ is the mass density of water and b the beam, and in this form it applies to any size of plate. The relation between K and the coefficients C_Δ and C_V may be shown as follows:

$$K = \frac{\Delta}{\frac{1}{2}\,\rho\,V^2\,b^2}$$

where $\rho = w/g$, the mass density of water, slugs/cu.ft.

$$\Delta = C_\Delta\,wb^3$$

$$V = C_V\sqrt{gb}$$

then $$K = \frac{C_\Delta\,wb^3}{\frac{1}{2}\,\frac{w}{g}\,C_V{}^2\,gb^3} = 2\,\frac{C_\Delta}{C_V{}^2}$$

When hulls of the same form but of different size are compared according to Froude's law, that is, with the values of C_Δ and C_V the same in the two cases, the value of K will also be the same. The converse, however, is not true, since a given value of K may correspond to any values of C_Δ and C_V that give the correct ratio of $2C_\Delta/C_V{}^2$. The use of this planing coefficient is thus confined to cases in which the effect of gravity, and therefore the limitations imposed by Froude's law, can be neglected. The points shown in figures 29 to 32 were calculated from the curves of figures 20 to 27 for a series of values of C_Δ and C_V. The scattering of the points is an indication of the validity of the assumptions involved in the development of the planing coefficient K. It may be seen that the scattering is chiefly confined to the points for small values of C_Δ, corresponding to a relatively high proportion of hydrostatic lift on the planing plate. The points for the higher values of C_Δ lie reasonably close to the mean curves, thus indicating that the curves may be used to determine the values of Δ/R, wetted length, and center of pressure for higher values of C_Δ and C_V than those covered by the present tests. Although the curves of figures 29 to 32 give only the results for the best trim angles, the same method may be used to extrapolate the test results for any other trim to values of load and speed greater than those tested.

EXAMPLES

The calculations for several typical cases will be given here in order to show the application of the results to design problems.

(1) The forebody of a flying boat having 20° dead rise carries a load of 10,000 pounds at 55 feet per second on a 7-foot beam. Required to find the lowest trim angle that will hold the wetted length to 7 feet on the longitudinally straight planing bottom, and the corresponding resistance and center of pressure. Using the original model results (figs. 11 to 14), we have $\Delta_m = \Delta_f \left(\frac{b_m}{b_f}\right)^3$ where the subscript m represents model conditions and subscript f the full-scale conditions, then $\Delta_m = 10,000 \left(\frac{1.33}{7}\right)^3 = \frac{10000}{144.6} = 69.1$ lb.

$$V_m = V_f \sqrt{\frac{b_m}{b_f}} = V_f \sqrt{\frac{1.33}{7}} = \frac{55}{2.29} = 24.0 \quad \text{f.p.s.}$$

$$(\text{w.l.})_m = \frac{7}{7}\, b_m = 1.0 \times 16 \text{ in.} = 16 \text{ in.}$$

Interpolating between the wetted lengths for 60 pounds and 80 pounds at $\tau = 8^\circ$ and $V = 24.0$ feet per second (fig. 13) the wetted length for $\Delta = 69.1$ pounds is found to be 17.2 inches. The trim angle required to give a wetted length of 16 inches is thus slightly greater than 8°. A similar interpolation for $\tau = 10^\circ$ gives a wetted length of 12.6 inches. Interpolation between these two values shows the trim angle for a wetted length of 16 inches to be 8.5°. The values of R_m and $(\text{c.p.})_m$ for $\Delta = 69.1$ pounds are next found by interpolation for angles of 10° and 8°. Cross curves of these values, interpolated to $\tau = 8.5^\circ$, give

$$R_m = 12.0 \text{ lb.} \qquad (\text{c.p.})_m = 7.8 \text{ in.}$$

from which

$$R_f = 12.0 \left(\frac{b_f}{b_m}\right)^3 = 12.0 \times 144.6 = 1{,}740 \text{ lb.}$$

and

$$(\text{c.p.})_f = 7.8 \text{ in.} \left(\frac{b_f}{b_m}\right) = 7.8 \times 5.25 = 41.0 \text{ in. or } 3.41 \text{ ft.}$$

(2) The forebody of a flying boat having 20° dead rise carries a load of 10,000 pounds at 55 feet per second on a 7-foot beam. Required to find the wetted length of longitudinally straight planing bottom when the hull is running at the best trim angle, and the corresponding resistance, trim angle, and center of pressure

$$C_\Delta = \frac{\Delta}{wb^3} = \frac{10000}{64 \times 7^3} = \frac{10000}{21950} = 0.457$$

$$C_V = \frac{V}{\sqrt{gb}} = \frac{55}{\sqrt{32.2 \times 7}} = \frac{55}{15} = 3.67$$

For these values of C_Δ and C_V, we find from figures 24 and 25,

$$C_R = 0.072 \quad \frac{w.l.}{b} = 1.45 \quad \frac{c.p.}{b} = 0.72 \quad \tau_o = 6.4^\circ$$

then

$$R = 0.072\ wb^3 = 0.072 \times 21,950 = 1,600\ \text{lb.}$$

$$w.l. = 1.45 \times 7 = 10.15\ \text{ft.}$$

$$c.p. = 0.72 \times 7 = 5.04\ \text{ft.}$$

(3) A flat planing surface (0° dead rise) carries a load of 2,000 pounds at 30 feet per second. Required to find the beam necessary to give a wetted length of 2b when the surface is running at the best trim angle, and the corresponding resistance, wetted length, and center of pressure.

The beam is unknown, hence the coefficients C_Δ and C_V cannot be computed. A direct solution, however, may be obtained from figure 29. The value of K for $\frac{w.l.}{b} = 2$ is 0.112.

$$K = \frac{\Delta}{\frac{1}{2}\rho\ V^2\ b^2}, \text{ for sea water } \frac{1}{2}\rho = \frac{64}{2 \times 32.2} = 0.995$$

$$0.112 = \frac{2000}{0.995 \times 30^2\ b^2} \qquad b^2 = \frac{2000}{0.995 \times 900 \times 0.112} = 20$$

$$b = 4.47\ \text{ft.}$$

$$\text{for } K = 0.112, \quad \frac{\Delta}{R} = 7.02 \quad \text{and} \quad \frac{c.p.}{b} = 1.40$$

$$R = \frac{2000}{7.02} = 286\ \text{lb.} \qquad c.p. = 4.47 \times 1.40 = 6.26\ \text{ft.}$$

This solution may now be checked, using the known beam by means of figures 20 and 21.

$$C_\Delta = \frac{\Delta}{wb^3} = \frac{2000}{64 \times 89.5} = 0.35 \quad C_V = \frac{V}{\sqrt{gb}} = \frac{30}{12} = 2.5$$

for these values of C_Δ and C_V in figures 20 and 21.

$$C_R = 0.0493 \qquad \frac{w.l.}{b} = 2.15 \qquad \frac{c.p.}{b} = 1.45$$

$$R = 0.0493\ wb^3 = 283\ \text{lb.} \qquad w.l. = 2.15\ b = 9.61\ \text{ft.}$$

$$c.p. = 1.45\ b = 6.48\ \text{ft.}$$

(4) The forebody of a seaplane float having 30° dead rise carries a load of 2,000 pounds at 50 feet per second on a beam of 3.5 feet. Required to find the best trim angle, and the corresponding resistance, wetted length and center of pressure at best angle.

$$C_\Delta = \frac{2000}{64 \times 3.5^3} = 0.726 \qquad C_V = \frac{50}{\sqrt{32.2 \times 3.5}} = 4.72$$

This value of C_Δ is outside the range tested, hence the curves of figure 32 will be used.

$$K = \frac{2000}{0.995 \times 2500 \times 12.2} = 0.066$$

for this value of K

$$\frac{\Delta}{R} = 5.15 \qquad \frac{w.l.}{b} = 1.53 \qquad \frac{c.p.}{b} = 0.74$$

$$R = 388\ \text{lb.} \qquad w.l. = 5.36\ \text{ft.} \qquad c.p. = 2.59\ \text{ft.}$$

CONCLUDING REMARKS

The test results presented in this paper should aid the designers of seaplanes in making preliminary calculations for new designs, and improve the general understanding of the phenomena encountered in the planing condition. The designer should bear in mind that the measurements given are strictly applicable only to bottoms of the forms tested, running in undisturbed water. The forces recorded include the water reaction and such air forces as arise from the interference between the bottom and the water surface. The air drag of the boat hull must be estimated separately in order to arrive at the total resistance as measured in hull-model tests.

The characteristics of a hull at speeds below the lower limit of planing can be determined only by actual model tests. The magnitude of the hump resistance, however, can probably be estimated reasonably well for a hull having a long flat on the planing bottom, since the hump usually occurs at speed coefficients within the range covered by these tests.

Further work on planing surfaces is included in the research program of the N.A.C.A. tank. Surfaces with longitudinal curvature and with arched cross sections will be tested in the near future. Theoretical work on planing phenomena, particularly on the form of the wake acting on the afterbody of a stepped boat, is needed. The appendix to this paper offers a solution to this problem for a simple case. It is hoped that the experiments reported in the present paper, together with the work done by Sottorf and others, will lead to a more scientific foundation for future design work.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 7, 1934.

APPENDIX

WAKE OF FLAT SURFACE

The experimental results presented in this paper offer a basis for calculating the performance of an unstepped planing surface, or of the forebody of a stepped hull. Estimating the forces on the afterbody of a flying boat or hydroplane, however, is complicated by the fact that the planing surface is working in the disturbed wake of the forebody. The wake of a V-bottom planing surface has a complex form that changes considerably with changes in the speed, load, and trim angle. Sottorf has measured the contours of the wave formation for several planing surfaces, and gives a good description of their general characteristics in reference 3. The procedure involved in such measurements is so laborious, however, that complete experiments to determine the wake profiles for all the cases covered in the present paper are not practicable.

At combinations of the independent variables that cause the surface to be supported primarily by dynamic lift, the wake of a flat-bottom surface approaches a fairly simple form. It takes the approximate shape of a rectangular trough-like depression for a considerable distance aft of the trailing edge of the surface. A theoretical solution of the wake profile for such a case is offered here in the hope that it will be of some use in itself, and may lead to formulas for calculating the shape of the wake for less simple conditions.

Referring to figure 33, it is assumed that the hydrostatic lift is equal to the weight of water displaced by that portion of the surface extending below the free-water surface; then

$$\text{Hydrostatic lift} = \frac{w b d^2}{2 \tan \tau} \qquad (1)$$

where w is the weight-density of water, lb./cu.ft.

b, beam, ft.

d, draft, ft.

τ, trim angle.

Representing the total load by Δ and the dynamic lift by L

$$L = \Delta - \frac{w b d^2}{2 \tan \tau}, \qquad \frac{L}{\Delta} = 1 - \frac{w b d^2}{2\Delta \tan \tau} \tag{2}$$

If the effect of the static lift on the wake profile is neglected, and the wake ordinates measured below the base line a-a, the dynamic lift L can be equated to the downward momentum imparted to the water in unit time, thus

$$L = \rho S V u_1 \tag{3}$$

where ρ is the mass density of water, slugs/cu.ft.

S, an unknown area of apparent mass

V, the translational speed of the surface

u_1, the downward velocity of the water surface at station 1.

By this definition, S is an unknown cross-sectional area of water such that if all the water passing through this area in unit time were given a downward velocity of u_1 the momentum thus imparted would equal the dynamic lift. The subsequent solution depends upon the assumption that S remain the same for all stations along the wake. At any station a distance x behind the trailing edge of the surface the total kinetic plus potential energy of the wake is assumed to be constant, and equal to the kinetic energy at station 1, the after edge of the surface.

$$K E_1 = \frac{\rho S V u_1^2}{2} \tag{4}$$

The potential energy at any point is equal to the work required to depress the water surface a distance z against the hydrostatic head. The potential energy at any station x for unit time is thus

$$P.E. = \frac{w b V z^2}{2} \tag{5}$$

where b is the width of the trough, assumed to equal the beam of the surface. The total energy is

$$\frac{\rho S V u_1^2}{2} = \frac{\rho S V u^2}{2} + \frac{w b V z^2}{2} \tag{6}$$

where u is the downward velocity of the surface at x. From equation (3), $\rho\ S\ V\ u_1 = L$, hence (6) may be rewritten

$$L\ u_1 = \rho\ S\ V\ u^2 + w\ b\ V\ z^2 \tag{7}$$

Assuming that the speed of the wake relative to the planing surface equals V, we have

$$\frac{dz}{dx} = \frac{u}{V}, \qquad u = \frac{V\ dz}{dx}$$

Substituting in (7)

$$\rho\ S\ V^3 \left(\frac{dz}{dx}\right)^2 = L\ u_1 - w\ b\ V\ z^2 \tag{9}$$

$$\frac{dz}{\sqrt{L\ u_1 - w\ b\ V\ z^2}} = \frac{dx}{\sqrt{\rho\ S\ V^3}} \tag{10}$$

$$\frac{1}{\sqrt{w\ V\ b}} \int \frac{dz}{\sqrt{\frac{L\ u_1}{w\ b\ V} - z^2}} = \int \frac{dx}{\sqrt{\rho\ S\ V^3}} \tag{11}$$

$$\frac{1}{\sqrt{w\ b\ V}} \sin^{-1}\left(\frac{z}{\sqrt{\frac{L\ u_1}{w\ b\ V}}}\right) = \frac{x}{\sqrt{\rho\ S\ V^3}} + C_1 \tag{12}$$

$$z = \sqrt{\frac{L\ u_1}{w\ b\ V}} \sin\left[\left(\frac{x\sqrt{w\ b\ V}}{\sqrt{\rho\ S\ V^3}}\right) + C_1\sqrt{w\ b\ V}\right] \tag{13}$$

$$z = 0 \quad \text{when} \quad x = 0, \quad \text{hence} \quad C_1 = 0$$

Assuming that the wake surface at station 1 is parallel to the planing bottom, we may substitute $V \tan \tau = u_1$ in (13) to give

$$z = \sqrt{\frac{L \tan \tau}{w\ b}} \sin\left(\frac{x\sqrt{w\ b}}{\sqrt{\rho\ S\ V^2}}\right) \tag{14}$$

From equation (3) $L = \rho\ S\ V\ u_1 = \rho\ S\ V^2 \tan\tau$, then

$$\rho\ S\ V^2 = \frac{L}{\tan\tau} \tag{15}$$

Making this substitution in (14) to eliminate the unknown S, we have

$$z = \sqrt{\frac{L \tan\tau}{w\ b}}\ \sin\left(x\sqrt{\frac{w\ b \tan\tau}{L}}\right) \tag{16}$$

Equation (16) shows that, for the limiting case corresponding to no hydrostatic lift, the wake has the form of a sine curve, and is independent of the speed. The value of z_{max} is found by equating

$$\frac{dz}{dx} = 0 \text{ as}$$

$$z_{max} = \pm\sqrt{\frac{L \tan\tau}{w\ b}} \tag{17}$$

and the value of x corresponding to z_{max} is

$$x_{z_{max}} = \frac{\pi}{2}\sqrt{\frac{L}{w\ b \tan\tau}} \tag{18}$$

As a check on the assumptions involved in the foregoing derivation, the calculated wake forms are compared with Sottorf's measurements (reference 3) of the wake behind a flat surface of 0.30-meter (0.985-foot) beam carrying 18 kilograms (39.7 pounds) at a speed of 6 meters (19.7 feet) per second, for three different trim angles. The curves are shown in figure 34. It may be seen that neglecting the effect of hydrostatic lift causes a serious error in the calculated wake form for conditions corresponding to values of L/Δ (hydrodynamic-lift/total-load, equation (2)), appreciably below 1.0. As the value of L/Δ approaches unity, however, the agreement becomes reasonably good. It is hoped that further work on the problem will lead to a solution taking account of the effect of hydrostatic lift, and possibly to an extension covering the form of the wake behind V-bottom surfaces.

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TABLE I. Test Data for Model 27, 0° Dead Rise

Water density 63.5 lb./cu.ft. Water temperature 61° F. Kinematic viscosity = 0.0000123 ft.^{2}/sec.

Trim angle, τ = 2°

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
5	20.7	0.8	0.3	4	0.4
	26.4	.8	.3	2	.3
	31.2	.8	5.1	1	.3
	36.0	.7	4.9	1	.2
10	20.5	1.4	10.8	7	0.4
	20.7	1.5	4.8	6	.5
	26.4	1.2	4.6	4	.3
	31.3	1.3	4.7	4	.4
	37.0	1.3	4.7	3	.4
20	20.2	3.7	16.1	17	0.8
	20.7	3.5	13.8	18	.8
	26.3	2.6	5.9	8	.4
	31.3	2.4	4.6	6	.5
	36.5	2.5	3.4	5	.2
	46.5	2.6	2.5	3	.3
40	25.1	8.1	24.1	30	1.0
	25.7	7.4	21.0	30	.9
	26.2	6.6	17.0	21	.8
	30.0	6.4	–	14	.4
	31.2	5.7	10.2	13	.6
	37.0	4.6	4.9	9	.4
	46.5	5.2	4.3	7	.6
60	25.2	11.8	33.0	44	1.8
	30.2	11.5	25.0	32	1.1
	33.8	10.4	17.7	23	.7
	35.6	8.6	10.8	16	.5
	36.5	8.2	9.6	13	.6
	39.1	8.8	10.1	16	.5
	44.1	8.4	7.6	12	.4
	46.4	7.0	4.5	9	.4
80	36.0	14.3	21.9	30	1.1
	39.0	13.2	17.6	–	.8
	43.5	12.3	11.9	18	.6
	44.2	11.8	–	16	.5
	46.0	10.4	7.6	12	.5

Trim angle, τ = 4°

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
5	14.3	0.7	2.2	4	0.0
	16.7	.6	1.9	3	.2
	18.8	.7	2.1	2	.4
	21.3	.6	1.9	2	.2
	25.8	.5	1.6	2	.0
	30.6	.6	1.9	2	.1
10	14.3	1.2	6.6	7	0.2
	16.8	1.2	3.1	5	.3
	19.0	1.2	3.1	3	.3
	20.8	1.2	3.1	3	.2
	25.8	1.2	5.4	3	.1
	30.6	1.2	5.4	3	.1
20	14.2	2.7	12.7	16	0.8
	14.8	2.6	–	14	.7
	16.8	2.4	7.2	10	.5
	18.9	2.3	5.3	7	.4
	20.7	2.2	2.9	6	.2
	25.9	2.2	3.0	4	.3
	30.5	2.2	2.3	4	.1
	36.0	2.2	4.1	3	.1
40	14.7	5.6	23.4	36	2.0
	17.0	5.7	19.5	28	1.7
	19.0	5.3	–	18	1.2
	19.0	5.3	15.5	20	.5
	20.7	5.0	10.6	14	.6
	25.9	4.3	5.3	8	.3
	30.4	4.1	2.8	6	.1
	35.6	4.3	3.4	5	.2
	45.6	4.4	2.9	4	.3
60	17.4	8.5	27.2	42	2.8
	19.4	8.5	24.5	34	2.6
	21.1	8.2	19.4	26	1.6
	25.8	7.2	9.8	14	.7
	30.9	6.5	5.2	8	.4
	36.1	6.2	3.4	6	.2
	45.6	6.4	2.9	5	.3
80	19.4	11.4	29.9	44	3.3
	21.0	11.4	27.0	38	2.4
	30.6	9.1	8.9	11	.7
	36.2	8.2	5.2	7	.3
	45.8	8.0	5.2	6	–

Trim angle, τ = 6°

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
5	13.4	0.8	2.1	2	0.1
	15.2	.7	1.8	2	.1
	17.9	.7	1.8	2	.1
	19.3	.8	2.0	2	.4
	21.3	.7	1.8	2	.1
10	13.4	1.4	1.8	4	0.1
	15.3	1.3	1.7	3	.1
	18.0	1.4	1.8	3	.2
	19.3	1.4	1.8	3	.5
	21.2	1.3	1.7	2	.5
	26.4	1.4	1.8	2	.1
20	13.0	2.9	11.0	12	0.9
	13.1	2.8	7.6	10	.6
	15.0	2.7	5.2	7	.3
	17.8	2.6	2.7	4	.3
	19.3	2.6	2.8	4	.3
	21.2	2.5	1.5	3	.3
	26.4	2.5	1.5	3	.1
40	13.0	6.1	18.1	28	2.6
	15.0	5.8	14.3	22	1.6
	17.6	5.4	7.9	11	.9
	19.6	5.2	5.2	8	.6
	21.6	5.1	4.0	6	.4
	25.9	4.9	2.7	5	.3
	31.2	5.0	2.7	3	.1
60	15.0	9.0	21.7	32	2.8
	17.5	8.6	16.6	24	2.0
	17.5	8.8	16.4	22	2.4
	19.4	8.5	11.8	16	1.6
	21.3	8.0	8.4	11	.7
	26.8	7.5	3.9	6	.3
	31.0	7.2	2.7	5	.2
	34.2	7.3	2.7	4	.2
80	15.0	12.0	26.1	42	3.9
	17.6	12.1	22.1	32	3.3
	19.7	11.8	17.8	24	2.8
	21.3	11.2	11.1	18	1.5
	26.6	10.1	6.3	9	.6
	31.2	9.7	5.0	7	.5
	34.2	9.5	5.0	5	.3

Trim angle, τ = 8°

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
5	12.0	0.9	1.9	2	0.1
	14.3	.9	1.9	2	.1
	16.5	.8	1.7	1	.1
	18.5	.8	1.6	1	.6
10	12.1	1.6	5.2	4	0.2
	14.0	1.6	1.7	4	.1
	16.6	1.6	.5	3	.1
	18.4	1.6	.4	2	.5
	20.7	1.5	2.7	2	.3
20	12.2	3.2	5.2	9	0.6
	14.1	3.2	5.2	7	.2
	16.6	3.0	2.7	4	.2
	18.5	3.0	2.7	3	.4
	20.5	3.0	1.4	3	.4
40	12.2	6.9	14.4	23	2.2
	13.0	7.0	–	19	2.2
	13.8	6.7	11.5	17	1.5
	16.5	6.3	6.4	9	.7
	19.0	6.1	3.9	7	.6
	20.5	5.9	3.9	6	.1
	25.5	6.1	1.6	3	.2
60	13.1	10.4	19.4	30	3.6
	13.8	10.2	18.4	30	2.9
	15.0	10.3	–	22	2.6
	16.3	9.8	12.3	18	1.7
	18.8	9.4	8.0	12	1.2
	20.0	9.3	6.4	9	.8
	25.4	8.8	3.9	5	.4
	30.3	8.7	2.6	3	.3
80	13.3	14.0	23.1	38	4.7
	15.2	13.8	20.6	31	3.5
	16.3	13.5	18.4	26	2.8
	17.6	13.7	–	21	2.5
	18.7	13.1	12.6	18	1.8
	20.0	12.8	10.2	15	1.5
	25.8	12.0	5.1	7	.5
	30.0	11.5	5.0	5	.2

[1]Measured from trailing edge.

TABLE II. Test Data for Model 28, 10° Dead Rise

Water density 63.5 lb./cu.ft. Water temperature 61° F.

Kinematic viscosity = 0.0000123 ft.^{2}/sec.

Trim angle, $\tau = 3^\circ$					
Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
5	20.9	1.4	6.0	19	1.0
	21.0	1.1	3.3	16	.9
	25.3	1.5	4.6	15	.8
	26.6	1.2	3.3	14	.8
	36.2	1.4	2.0	12	.5
	36.3	1.4	2.0	12	.5
	46.1	1.8	4.8	8	.5
	46.8	1.1	-	8	.4
10	20.8	2.5	11.0	25	1.3
	21.0	2.4	11.0	-	1.1
	21.1	2.3	10.8	20	1.1
	25.3	2.4	8.7	21	.9
	26.3	2.5	9.9	20	.8
	35.8	2.6	5.9	16	.9
	36.8	2.8	8.4	14	.6
	46.0	2.9	9.6	12	.6
	47.4	3.1	4.6	11	.6
20	20.7	4.4	17.9	36	1.7
	21.1	4.3	18.0	34	1.5
	25.7	4.4	11.7	26	1.1
	26.3	4.7	14.0	28	1.5
	35.3	5.4	10.2	22	1.1
	37.5	5.2	8.7	20	.9
	47.2	5.7	7.5	16	.8
40	21.1	8.3	31.0	55	2.2
	22.4	8.6	30.7	52	2.2
	26.1	6.3	26.0	45	1.9
	36.3	10.2	17.4	-	1.0
	36.5	9.4	14.1	29	1.2
	36.5	8.1	11.5	27	1.3
	46.7	10.0	10.2	20	1.0
60	35.6	12.9	18.8	40	1.5
	36.6	13.9	20.7	50	1.5
	37.5	13.4	18.7	38	1.5
	45.4	14.4	12.6	27	1.1
	46.0	14.3	18.0	30	1.2
Trim angle, $\tau = 4^\circ$					
5	14.0	0.7	3.1	9	0.6
	15.5	.7	6.1	7	.7
	17.8	.7	6.1	8	.8
	18.8	.8	6.2	7	.8
	20.0	.8	6.2	6	.5
	21.5	.7	4.0	6	.5
	22.1	.9	6.9	6	.6
	22.4	.8	4.3	6	.7
	27.0	.7	6.3	6	.5
	27.2	.9	8.6	8	.5
	27.6	.5	3.5	5	.5
	38.0	.7	6.4	5	.4
	38.4	.5	-	4	.3
	47.4	.8	6.6	4	.1

Trim angle, $\tau = 4^\circ$					
Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
10	14.1	1.1	5.3	15	0.9
	15.4	1.1	8.0	12	.8
	17.6	1.1	9.1	8	1.1
	18.9	1.2	5.1	10	.9
	20.0	1.2	8.1	8	.8
	21.6	1.4	5.3	9	.8
	21.9	1.4	9.5	8	.6
	23.0	1.4	5.2	8	.7
	27.3	1.4	9.5	8	.5
	28.1	1.3	3.2	8	.5
	37.3	1.5	9.6	6	.3
	38.5	1.2	1.9	5	.3
	44.5	1.5	3.8	6	.6
	47.3	1.6	9.7	4	.4
20	13.9	2.3	13.5	25	1.6
	15.4	2.5	13.4	22	1.3
	17.6	2.3	14.4	18	1.4
	18.8	2.3	7.7	15	1.2
	21.3	2.4	6.6	13	.9
	23.1	2.5	5.5	12	.8
	27.9	2.4	5.0	10	.6
	38.6	2.6	3.7	8	.8
	44.0	2.7	4.2	9	.7
40	17.8	5.4	19.8	36	2.4
	19.6	5.4	-	29	1.4
	19.7	5.6	17.1	26	1.8
	21.5	5.3	16.6	28	1.7
	21.6	5.4	14.1	24	1.6
	23.3	5.1	10.8	19	1.4
	27.9	4.9	7.2	14	1.0
	38.5	4.8	4.8	11	.9
	44.7	5.4	1.3	11	1.0
60	18.2	8.4	26.0	50	2.9
	19.7	8.5	25.0	41	2.8
	19.9	8.6	24.5	40	2.8
	21.2	8.7	23.9	37	2.5
	21.6	8.5	22.1	40	2.4
	28.1	7.6	10.7	20	1.2
	38.9	7.1	5.8	13	.9
	44.2	8.0	5.6	14	1.0
80	19.3	11.4	29.5	52	3.5
	20.0	11.5	29.4	50	3.5
	21.0	11.6	28.4	45	3.1
	22.1	11.7	27.2	50	3.2
	26.9	11.1	18.5	30	2.1
	39.0	9.3	7.3	16	1.0
	48.0	9.9	5.7	13	1.0

[1]Measured from trailing edge.

TABLE II. Test Data for Model 28, 10° Dead Rise (Continued)

Water density 63.5 lb./cu.ft. Water temperature 61° F.

Kinematic viscosity = 0.0000123 ft.^{2}/sec.

Trim angle, τ = 6°					
Load lb.	Speed f.p.s.	Resist-ance lb.	[1]Center of pres-sure in.	Wetted length in.	Draft in.
5	13.2	0.6	3.8	6	0.5
	15.2	.6	5.8	6	.5
	15.2	.9	2.6	6	.5
	17.7	.7	2.0	5	.6
	20.4	.8	4.2	4	.6
	22.3	.6	3.7	3	.4
	27.2	.6	1.7	3	.4
	38.0	.6	3.5	2	.3
	46.1	.4	3.1	1	.2
10	12.2	1.4	5.4	10	0.9
	13.1	1.2	9.9	10	.7
	15.2	1.4	4.0	7	.8
	15.3	1.3	9.0	10	.6
	17.8	1.3	10.0	7	.8
	19.8	1.4	3.0	6	.7
	22.5	1.2	2.6	5	.6
	27.0	1.1	2.5	5	.5
	37.7	1.1	2.4	3	.4
	46.0	.8	2.2	2	.4
20	12.1	2.6	11.3	19	1.7
	14.0	2.6	7.5	14	1.3
	14.4	2.5	14.5	16	1.1
	18.0	2.5	4.8	9	1.0
	19.9	2.6	4.3	8	.8
	22.6	2.6	3.8	7	.8
	27.4	2.6	2.8	6	.6
	38.2	2.6	2.7	5	.6
	46.5	2.4	2.0	4	.5
40	13.2	5.7	18.7	32	2.3
	14.1	5.6	16.9	28	2.7
	14.3	5.7	17.9	28	2.5
	16.2	5.5	16.3	25	1.8
	17.8	5.5	10.2	17	1.8
	20.2	5.2	7.4	13	1.4
	22.6	5.0	5.4	10	.9
	27.1	4.8	4.6	9	1.0
	37.7	4.8	3.2	7	.7
	46.8	5.2	3.0	6	.6
60	13.2	8.7	23.4	42	3.9
	16.2	8.8	20.0	35	3.0
	18.7	8.6	17.1	28	2.3
	19.5	8.4	–	24	2.0
	19.7	8.6	14.1	23	2.5
	22.3	7.9	9.6	17	1.5
	27.7	7.4	5.4	11	1.1
	38.2	7.6	3.6	8	.8
	47.4	7.4	3.0	7	.7
80	16.1	11.7	25.7	50	4.1
	18.9	11.9	22.5	38	3.5
	19.4	11.9	20.6	32	3.0
	21.3	11.8	18.1	28	2.7
	27.5	10.1	7.8	15	1.5
	27.6	10.2	7.3	15	1.4
	32.0	9.8	5.0	11	1.1
	38.0	9.8	4.4	10	.9
	47.4	10.0	4.4	8	.9

Trim angle, τ = 8°					
Load lb.	Speed f.p.s.	Resist-ance lb.	[1]Center of pres-sure in.	Wetted length in.	Draft in.
5	13.0	0.8	5.5	5	0.6
	13.1	.8	7.9	5	.4
	15.2	.8	1.8	4	.5
	15.4	.7	7.7	6	.3
	17.8	.8	1.6	3	.5
	20.0	.8	1.7	3	.5
	21.7	1.0	2.1	3	.5
	25.2	.6	1.2	3	.3
	35.8	.8	3.7	2	.3
	45.5	.5	4.9	1	.1
10	13.0	1.5	3.7	7	0.8
	13.0	1.4	11.8	7	.6
	15.2	1.6	2.6	6	.7
	17.8	1.5	2.6	5	.7
	20.2	1.6	2.7	4	.7
	22.4	1.5	2.7	4	.5
	24.8	1.7	4.3	5	.5
	35.9	1.5	2.7	3	.3
	45.6	1.3	4.2	2	.3
20	13.0	3.0	6.2	12	1.2
	15.1	3.1	4.1	8	1.0
	17.6	3.0	3.0	7	1.1
	20.1	2.9	2.5	7	.9
	22.3	2.9	3.1	6	.7
	24.8	3.0	2.7	6	.7
	35.9	2.6	2.3	4	.5
	45.8	2.8	3.1	3	.5
40	13.0	6.5	14.1	24	2.8
	15.3	6.4	10.1	17	2.0
	18.2	6.1	5.6	11	1.4
	20.2	6.0	4.4	9	1.2
	22.2	5.9	3.9	8	.9
	24.8	5.6	3.1	8	1.0
	35.5	5.6	2.3	5	.6
	46.1	5.4	2.4	4	.5
60	12.9	10.1	19.3	35	3.4
	14.9	10.2	–	28	2.5
	18.1	9.5	10.6	17	2.2
	20.2	9.3	8.1	15	1.9
	22.5	8.8	5.5	13	1.2
	27.5	8.7	3.9	7	1.0
	35.8	8.5	3.0	6	.8
	47.0	8.7	2.7	5	.7
80	13.2	13.3	23.0	44	4.5
	15.0	13.7	21.5	38	3.7
	17.4	13.6	13.6	29	2.6
	19.2	13.0	–	24	1.8
	20.7	12.8	11.7	19	2.3
	22.4	12.6	8.6	15	1.7
	27.0	12.0	5.5	9	1.1
	35.6	11.6	3.6	8	.9

[1]Measured from trailing edge.

TABLE III. Test Data for Model 29, 20° Dead Rise

Water density 63.5 lb./cu.ft. Water temperature 60° F. Kinematic viscosity = 0.0000125 ft.²/sec.

Trim angle, τ = 4°

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
5	14.5	0.9	8.9	18	1.3
	29.7	1.3	5.0	9	.8
	35.7	1.6	3.9	7	.7
	45.1	1.7	4.1	6	.7
10	14.5	1.8	10.7	24	1.7
	30.0	2.2	6.1	13	1.0
	36.0	2.6	4.6	10	.8
	44.5	2.9	4.0	8	1.0
20	14.6	3.6	16.5	35	2.5
	30.2	4.6	3.9	17	2.4
	35.6	4.7	7.3	13	1.2
	45.0	5.3	5.9	11	1.1
40	30.1	8.5	11.0	24	2.0
	37.2	9.1	10.5	22	1.5
	45.6	9.8	7.4	18	1.4
60	30.3	11.6	13.8	33	2.3
	35.9	12.9	13.0	27	2.0
	45.1	14.0	9.1	24	1.7
80	30.4	15.2	18.5	37	2.7
	35.3	16.0	18.3	29	2.3

Trim angle, τ = 6°

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
5	17.2	1.0	7.0	9	1.0
	19.2	1.0	6.9	8	1.2
	21.2	1.0	5.1	6	.8
	23.9	1.0	6.8	6	.8
	29.5	.9	4.7	4	.7
	34.6	.9	4.6	4	.5
	44.5	1.0	5.0	2	.5
10	12.3	1.6	9.4	16	1.2
	17.1	1.7	5.6	12	1.4
	19.3	1.8	6.7	11	1.4
	21.0	1.7	5.6	10	1.0
	24.1	1.8	5.6	8	1.0
	29.5	1.8	4.7	6	1.0
	34.5	1.8	4.7	6	.8
	44.6	1.9	4.8	5	.6
20	12.3	2.9	13.4	26	2.6
	14.3	3.0	10.9	21	2.2
	17.0	2.8	8.2	17	1.8
	19.1	3.3	7.5	15	1.7
	21.0	3.4	7.0	14	1.5
	24.0	3.3	6.4	12	1.3
	29.2	3.6	5.3	9	1.2
	34.8	3.6	4.7	8	1.1
	44.1	3.6	4.7	7	1.1
40	14.4	6.4	17.6	34	3.6
	17.1	6.4	13.7	27	2.9
	19.4	6.4	11.3	23	2.4
	21.3	6.2	9.1	20	2.1
	24.4	6.2	8.2	17	1.9
	25.9	6.7	7.2	18	1.7
	29.3	6.9	6.9	14	1.7
	30.1	7.0	6.6	15	1.6
	34.4	6.9	6.6	12	1.4
	44.6	7.4	4.7	10	1.2
60	21.0	9.6	14.2	28	2.9
	24.5	9.4	10.5	22	2.4
	29.0	9.7	8.0	17	1.9
	35.0	10.6	7.3	15	1.7
	44.9	11.2	6.2	14	1.6
80	24.5	13.0	14.0	26	2.9
	29.1	12.9	10.0	21	2.4
	34.8	12.9	8.2	18	2.2
	43.0	14.4	6.9	16	1.8

Trim angle, τ = 8°

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
5	12.7	1.0	6.4	8	1.3
	14.8	1.1	6.6	7	1.0
	17.4	1.0	6.4	6	1.2
	19.3	1.1	6.8	5	1.1
	21.2	1.0	6.5	5	1.0
	25.2	1.0	4.5	3	.7
	30.4	.9	6.5	3	.5
	35.8	1.0	6.7	2	.7
	45.8	.8	8.0	1	.3

Trim angle, τ = 8°

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
10	12.7	1.7	7.0	11	1.7
	14.8	1.7	6.1	10	1.5
	17.2	1.8	5.1	8	1.6
	19.3	1.8	5.3	7	1.4
	21.1	1.6	5.0	7	1.1
	25.1	1.9	6.0	6	.8
	30.0	1.8	6.5	5	1.1
	36.0	1.8	5.4	4	.9
	45.6	1.7	6.1	3	.7
20	12.6	3.2	9.2	17	2.4
	14.8	3.4	7.0	14	2.0
	17.1	3.3	6.5	11	2.0
	19.2	3.5	5.7	10	1.8
	21.0	3.4	5.2	10	1.6
	25.1	3.5	5.2	9	1.1
	30.0	3.5	5.4	7	1.2
	35.6	3.6	4.9	6	1.1
	45.6	3.6	5.0	4	.8
40	12.6	6.7	15.3	29	4.1
	14.8	7.2	12.6	23	3.1
	17.2	6.8	9.0	18	2.9
	19.4	6.7	7.4	15	2.3
	21.2	6.7	6.2	14	2.2
	25.0	7.0	6.1	13	1.7
	30.4	7.2	1.9	9	1.1
	36.4	7.2	5.2	8	1.4
	46.0	7.4	4.2	6	1.1
60	17.3	10.7	13.7	25	3.7
	19.2	10.2	11.4	21	3.2
	21.0	10.0	9.5	19	2.7
	25.2	10.0	6.8	16	2.2
	30.5	10.4	6.1	12	1.9
	36.0	10.3	5.3	10	1.7
	46.0	11.1	4.7	8	1.5
80	19.1	14.4	15.8	28	4.2
	21.0	14.2	12.7	23	3.5
	25.5	13.4	8.7	18	2.7
	29.9	13.6	6.7	14	2.1
	36.2	13.9	6.4	13	1.9
	43.8	14.2	6.2	10	1.6

Trim angle, τ = 10°

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
5	13.2	1.1	8.2	6	0.9
	15.2	1.1	-	5	.8
	17.9	1.1	1.8	4	.8
	19.5	1.1	2.6	4	.8
	24.2	1.1	3.6	4	.7
	34.5	1.0	7.4	2	.6
	44.4	1.1	11.4	1	.4
10	13.1	2.0	4.8	8	1.2
	15.1	2.1	1.9	7	1.1
	17.7	2.1	4.8	6	1.4
	19.6	2.1	2.8	5	.9
	24.2	2.1	4.9	5	.8
	34.6	2.0	3.7	3	.6
	44.2	1.9	7.4	2	.5
20	13.1	3.8	7.2	12	1.9
	15.1	4.0	4.5	10	1.7
	17.7	4.0	4.4	8	1.7
	19.3	4.0	3.4	8	1.3
	24.6	4.2	4.6	7	1.1
	34.5	4.0	3.5	4	.7
	45.0	4.0	4.4	4	.6
40	12.9	7.8	12.0	22	3.6
	15.1	8.0	8.0	16	2.8
	18.0	7.6	6.0	12	2.4
	19.3	7.6	5.7	11	2.2
	24.5	7.7	4.2	9	1.5
	34.8	7.9	4.0	7	1.3
	44.7	7.9	3.0	5	.9
60	14.9	12.2	14.2	25	3.9
	17.7	11.7	9.6	18	2.9
	19.6	11.5	7.8	16	2.7
	24.6	11.5	5.4	11	2.0
	34.5	11.3	4.3	8	1.4
	44.4	11.8	3.8	7	1.2
80	17.4	16.5	13.9	25	4.2
	19.4	15.6	11.7	21	3.7
	24.5	15.3	6.6	14	2.3
	34.6	15.2	5.8	10	1.5
	40.3	15.8	5.8	9	1.4

[1]Measured from trailing edge.

TABLE IV. Test Data for Model 30, 30° Dead Rise

Water density 63.5 lb./cu.ft. Water temperature 57° F.

Kinematic viscosity = 0.0000130 ft.^{2}/sec.

Trim angle, τ = 4°

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
5	11.3	0.9	13.4	32	2.2
	13.2	1.3	12.2	30	2.1
	15.8	1.4	10.4	26	2.0
	18.2	1.5	10.7	23	2.0
	20.0	1.8	11.3	21	2.0
	25.0	2.1	9.7	18	1.4
	30.0	2.4	8.6	16	1.3
	35.5	2.4	6.6	13	1.2
	45.0	3.1	22.6	10	.8
10	11.3	1.8	15.2	42	2.8
	13.2	2.2	14.6	38	2.7
	16.1	2.6	14.1	35	2.5
	17.8	3.0	14.5	32	2.4
	20.0	3.4	14.1	31	2.4
	25.0	3.8	13.5	25	1.9
	30.0	3.9	10.7	21	1.7
	35.7	4.5	9.3	19	1.5
	45.1	4.9	7.8	15	1.2
20	20.2	5.8	17.6	42	3.1
	24.6	7.0	16.0	37	2.6
	29.8	7.8	14.1	31	2.4
	35.9	8.2	12.3	25	1.4
	45.2	9.2	10.8	21	1.8
40	29.8	12.7	17.9	42	3.1
	35.8	14.6	16.4	37	2.0
	40.7	15.3	14.4	33	2.3

Trim angle, τ = 6°

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
5	10.8	1.0	7.8	22	2.3
	13.2	1.2	8.4	19	2.0
	15.6	1.2	8.4	16	1.8
	18.1	1.3	6.7	13	2.0
	20.0	1.4	6.8	12	2.0
	25.2	1.6	4.4	10	1.3
	30.4	1.2	6.3	8	1.0
	35.9	1.7	3.6	6	1.1
	45.2	2.2	4.6	5	1.0
10	11.1	1.8	12.3	29	3.1
	12.9	2.1	11.6	26	2.8
	15.7	2.0	10.5	22	2.6
	18.2	2.4	8.8	19	2.3
	20.0	2.6	8.2	18	2.2
	25.2	2.7	8.4	14	1.8
	30.2	2.6	6.2	11	1.6
	35.5	3.5	7.2	10	1.4
	36.4	3.0	6.7	8	1.3
20	11.3	3.3	15.4	38	4.0
	13.1	3.8	14.6	34	3.7
	15.9	3.9	13.2	30	3.2
	18.4	4.5	12.6	27	3.4
	20.2	4.5	11.6	24	2.9
	25.1	5.5	9.7	20	2.2
	30.6	5.2	8.0	17	2.1
	35.7	5.7	7.3	13	1.7
	44.9	6.3	5.6	10	1.3
40	13.4	7.0	19.1	47	4.9
	15.9	7.5	18.3	42	4.5
	18.2	8.1	17.5	38	4.3
	19.6	7.9	16.2	36	3.9
	20.4	8.3	15.0	34	3.7
	25.2	9.2	12.3	28	3.1
	31.2	9.5	10.8	24	2.6
	35.0	11.3	12.1	20	2.4
	45.5	11.7	8.2	15	1.9
60	19.8	11.6	19.9	45	4.6
	25.1	13.0	15.7	35	3.8
	29.0	13.6	13.3	30	3.3
	35.5	14.9	11.5	24	2.9
	40.8	16.1	10.2	20	2.5
80	25.2	16.4	19.3	41	4.4
	29.4	17.4	16.0	35	3.9

Trim angle, τ = 8°

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
5	12.1	1.1	7.8	14	2.0
	14.3	1.3	8.2	12	1.7
	17.1	1.2	7.1	9	1.6
	18.8	1.1	7.0	8	1.5
	20.5	1.2	6.1	8	1.4
	26.1	1.3	6.4	6	1.3
	30.5	1.3	8.2	5	1.0
	36.7	1.1	7.9	5	1.0
	46.3	1.4	6.8	4	.6
10	12.0	1.9	9.0	20	2.8
	14.3	2.0	9.2	17	2.4
	16.9	2.1	7.3	14	2.3
	18.2	2.1	7.2	12	2.5
	20.4	2.2	6.2	12	2.0
	26.6	2.1	6.2	9	1.4
	30.0	2.1	6.2	7	1.2
	36.7	2.3	6.5	6	1.1
	46.3	2.7	6.0	5	.9
20	11.9	3.6	12.6	28	3.8
	14.3	3.6	10.6	25	3.3
	16.7	3.7	9.6	20	2.9
	18.4	4.1	8.8	18	3.1
	21.2	4.1	7.9	15	2.6
	26.3	4.3	7.1	13	2.0
	30.7	4.3	6.5	11	1.8
	36.6	4.2	5.4	9	1.5
	46.3	4.8	4.7	7	1.1

[1]Measured from trailing edge.

TABLE IV. Test Data for Model 30, 30° Dead Rise (Continued)

Water density 63.5 lb./cu.ft. Water temperature 57° F.

Kinematic viscosity = 0.0000130 ft.^{2}/sec.

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
Trim angle, $\tau = 8^{\circ}$					
40	11.8	6.9	17.1	40	5.4
	14.4	7.3	15.9	34	4.8
	17.0	7.5	13.4	28	4.2
	18.4	7.6	12.0	26	4.0
	21.0	7.5	10.5	23	3.4
	21.0	7.6	10.7	22	3.4
	25.6	7.7	8.5	19	2.7
	26.6	8.6	8.6	18	3.0
	30.6	8.3	8.1	16	2.4
	36.8	8.8	10.7	12	2.1
	46.4	9.2	6.1	10	1.7
60	12.0	10.6	20.1	48	6.5
	13.8	10.6	19.6	44	6.1
	16.7	11.3	17.8	37	5.5
	18.7	11.5	16.0	33	4.9
	21.1	11.6	13.4	29	4.3
	26.2	12.3	10.8	22	3.5
	30.6	12.8	8.9	18	2.8
	36.0	13.1	8.0	15	2.6
	46.6	14.1	7.0	14	2.0
80	13.8	14.5	-	51	7.0
	17.4	15.1	20.5	43	6.0
	21.0	16.2	17.5	35	5.1
	26.4	16.2	12.5	26	3.8
	30.4	16.5	10.6	22	3.2
	36.2	17.5	9.0	18	2.8
Trim angle, $\tau = 10^{\circ}$					
5	10.9	1.2	6.7	14	2.0
	13.6	1.1	6.6	9	1.6
	16.3	1.1	6.6	7	1.4
	18.0	1.2	8.8	6	1.6
	20.6	1.1	7.5	5	1.4
	25.4	1.0	8.2	4	1.0
	30.4	1.1	8.5	4	.8
	35.8	1.1	6.7	3	.6
	45.4	1.3	9.0	2	.6
10	11.1	2.1	8.4	16	2.8
	13.5	2.1	7.4	12	2.3
	16.0	2.2	7.5	11	2.1
	17.9	2.1	7.4	9	1.9
	20.5	2.1	6.4	8	1.8
	25.6	2.2	5.5	6	1.3
	30.7	2.0	5.2	5	1.1
	36.3	2.5	6.7	4	1.1
	45.2	2.6	6.0	3	.9
20	11.1	3.9	11.2	22	3.9
	14.0	4.1	8.9	18	3.0
	16.1	4.1	8.4	14	2.6
	18.5	4.2	7.0	13	2.8
	20.4	4.2	7.3	12	2.4
	25.5	4.1	5.2	10	2.0
	30.0	4.3	5.4	8	1.6
	35.8	4.5	5.5	6	1.4
	45.3	4.9	5.2	5	1.2

Load lb.	Speed f.p.s.	Resistance lb.	[1]Center of pressure in.	Wetted length in.	Draft in.
Trim angle, $\tau = 10^{\circ}$					
40	11.3	7.6	14.9	32	5.6
	14.0	7.9	13.3	27	4.6
	16.3	7.9	11.2	22	3.8
	18.3	8.0	9.5	19	3.6
	20.6	8.0	8.2	17	3.3
	25.4	8.2	7.2	14	2.6
	30.0	8.3	6.2	12	2.2
	35.7	8.9	5.9	9	2.0
	45.4	8.8	5.0	8	1.5
60	12.3	11.7	17.7	39	6.6
	13.3	11.8	17.3	37	6.2
	13.7	12.0	17.0	36	6.1
	16.1	12.1	14.5	30	5.1
	18.3	12.1	12.4	25	4.6
	20.4	12.0	10.6	21	3.9
	24.8	12.0	8.4	18	3.3
	30.1	12.7	7.2	14	2.8
	35.6	13.1	6.3	12	2.4
	44.8	14.1	5.3	10	2.0
80	12.3	15.4	16.2	46	7.8
	15.0	16.1	19.2	40	6.8
	16.4	16.4	18.0	36	6.2
	18.1	16.6	16.2	32	5.5
	20.4	16.6	13.6	27	4.9
	24.6	16.4	10.2	21	3.8
	30.0	17.1	8.1	16	3.0
Trim angle, $\tau = 12^{\circ}$					
5	30.8	1.1	8.1	3	0.7
	37.0	.9	7.4	2	.5
10	30.9	2.4	5.2	4	.8
	37.3	2.5	6.4	3	.8
20	31.1	4.9	4.2	6	1.3
	37.0	4.9	4.8	5	1.0
40	31.3	9.3	5.1	9	1.9
	37.2	9.7	4.8	8	1.6
60	30.7	14.2	6.0	11	2.3
	36.4	14.3	5.0	10	2.0

[1]Measured from trailing edge.

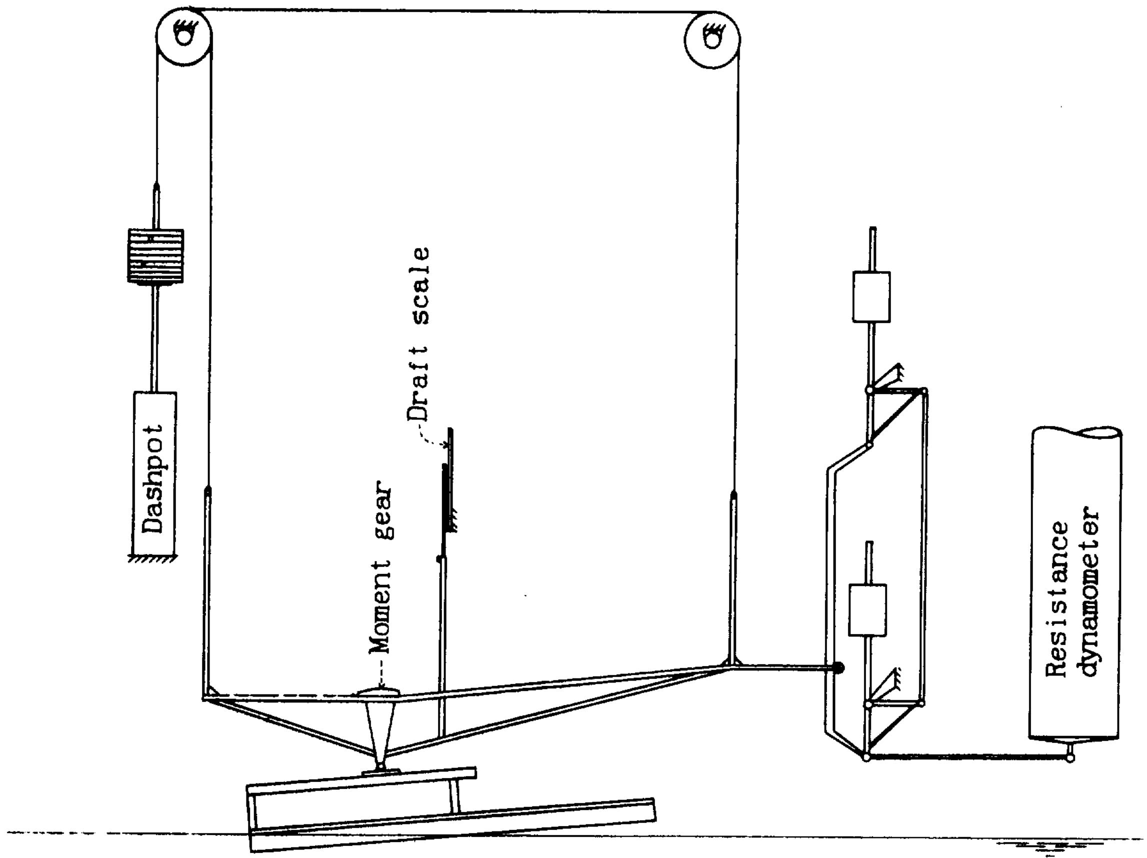

Figure 1. Diagram of towing gear.

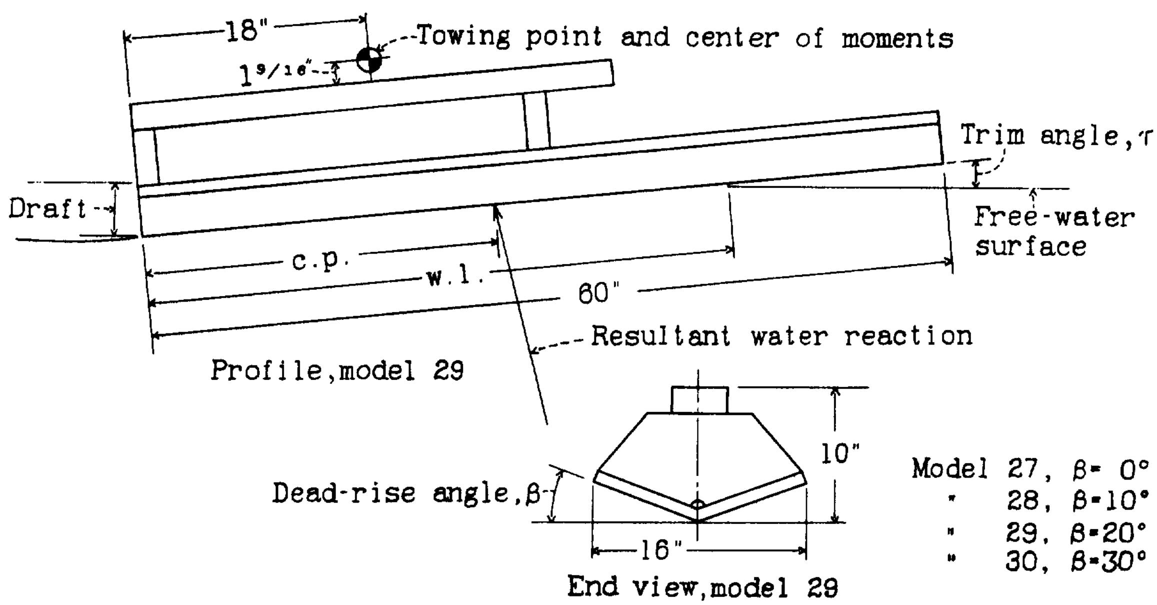

Figure 2.- Dimensions of planing surfaces.

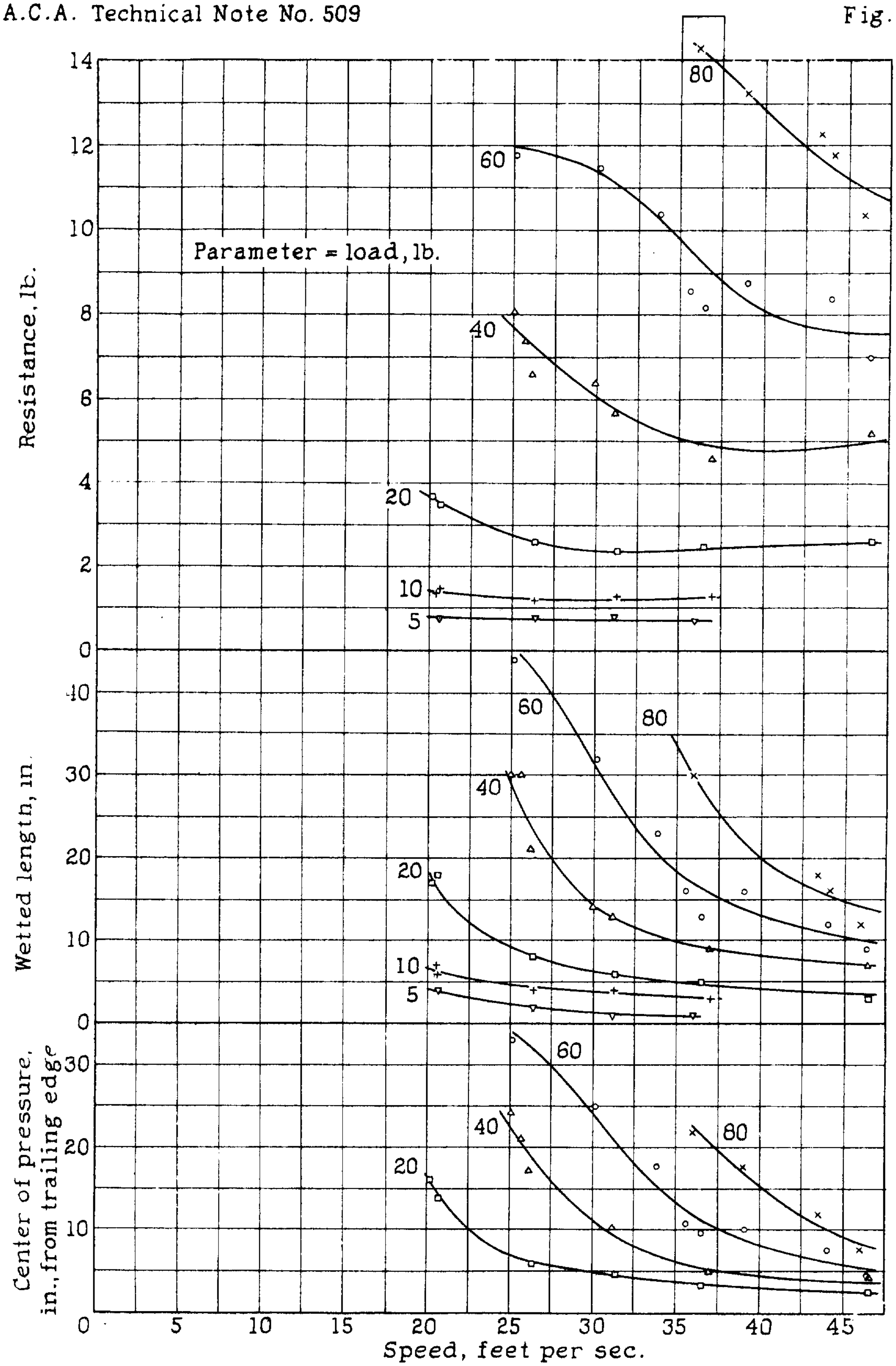

Figure 3. - Resistance, wetted length and center of pressure. Model 27, 0° dead rise. $\tau = 2°$.

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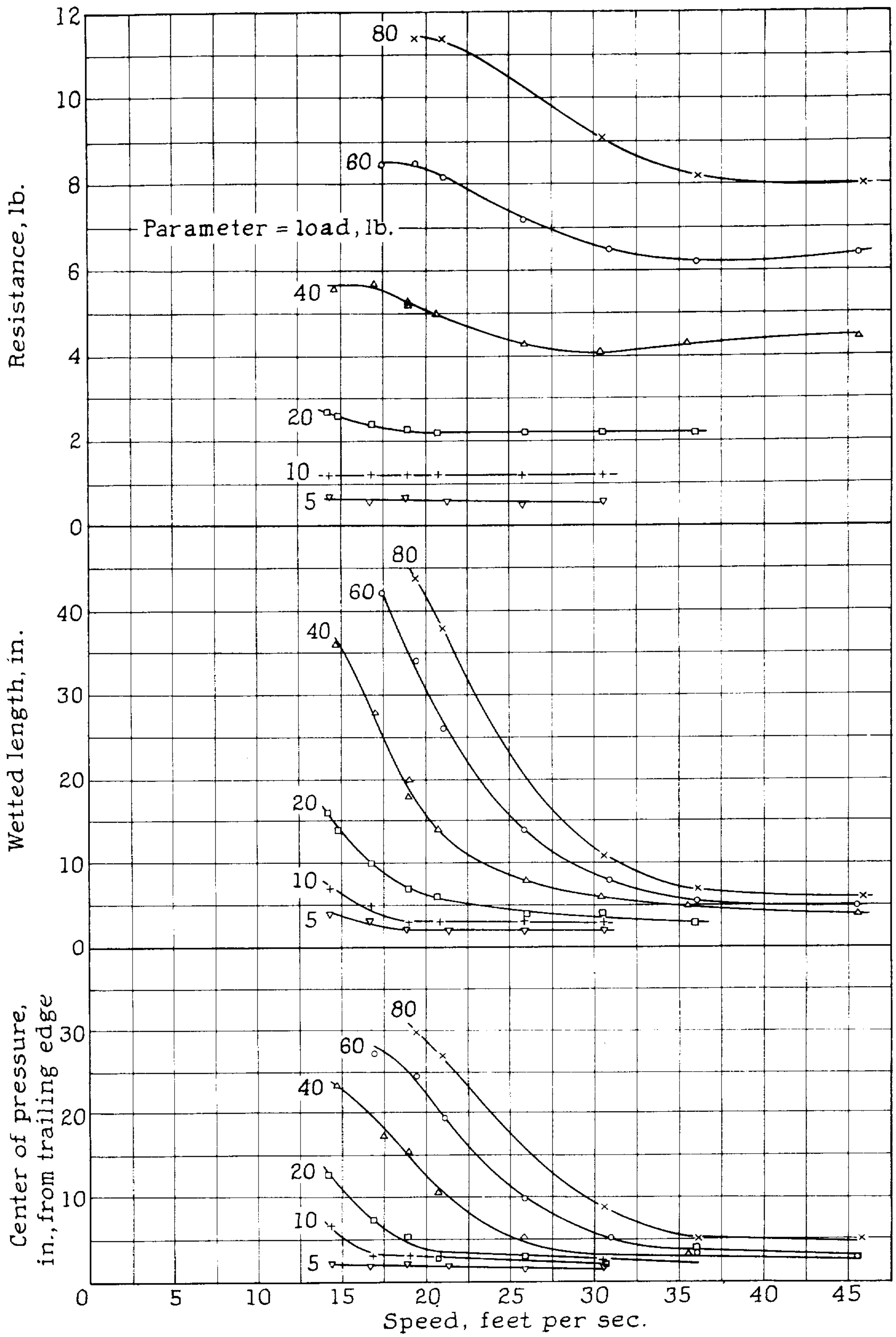

Figure 4. - Resistance, wetted length and center of pressure. Model 27, 0° dead rise. $\tau = 4°$

28

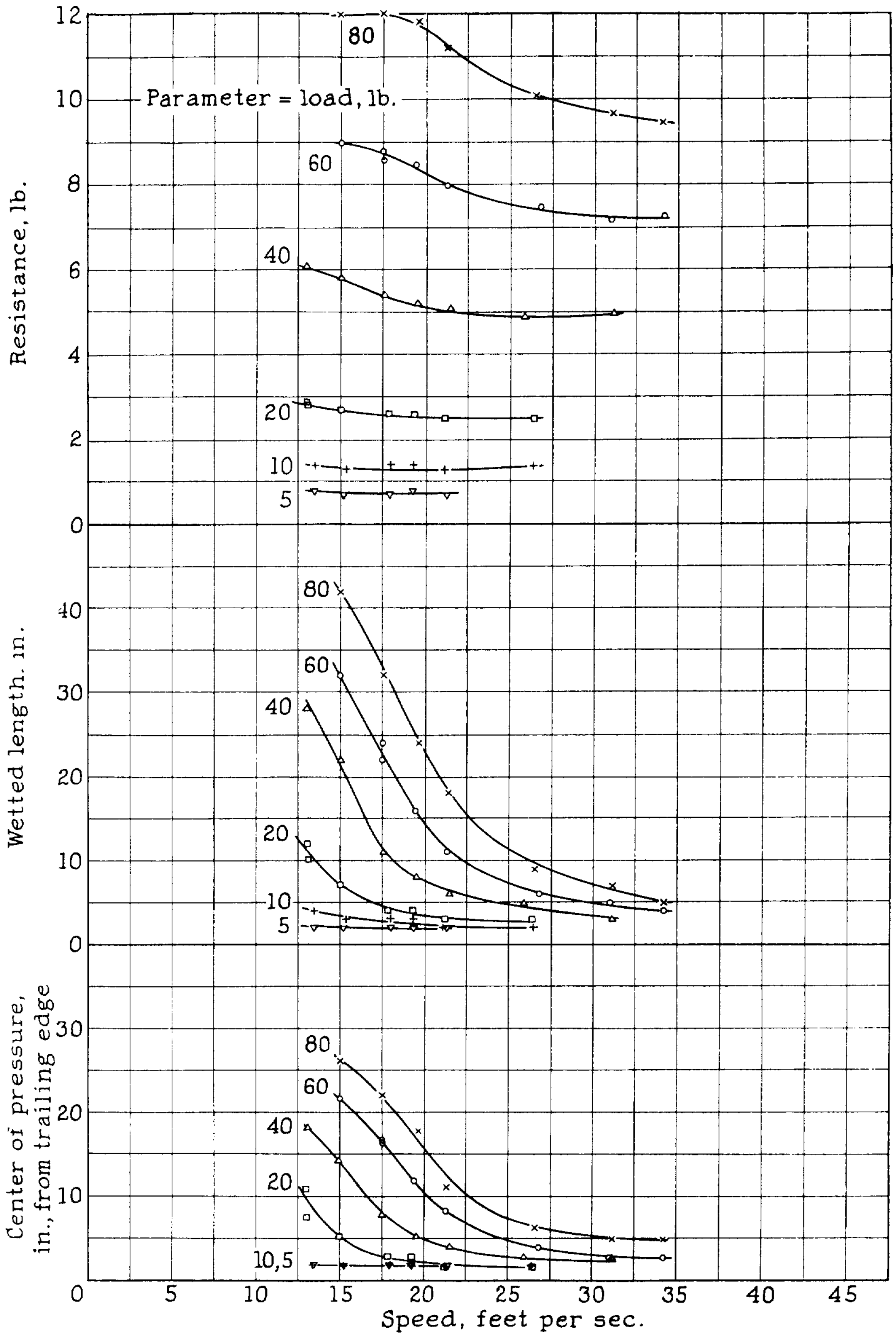

Figure 5. - Resistance, wetted length and center of pressure. Model 27, 0° dead rise. $\tau = 6°$

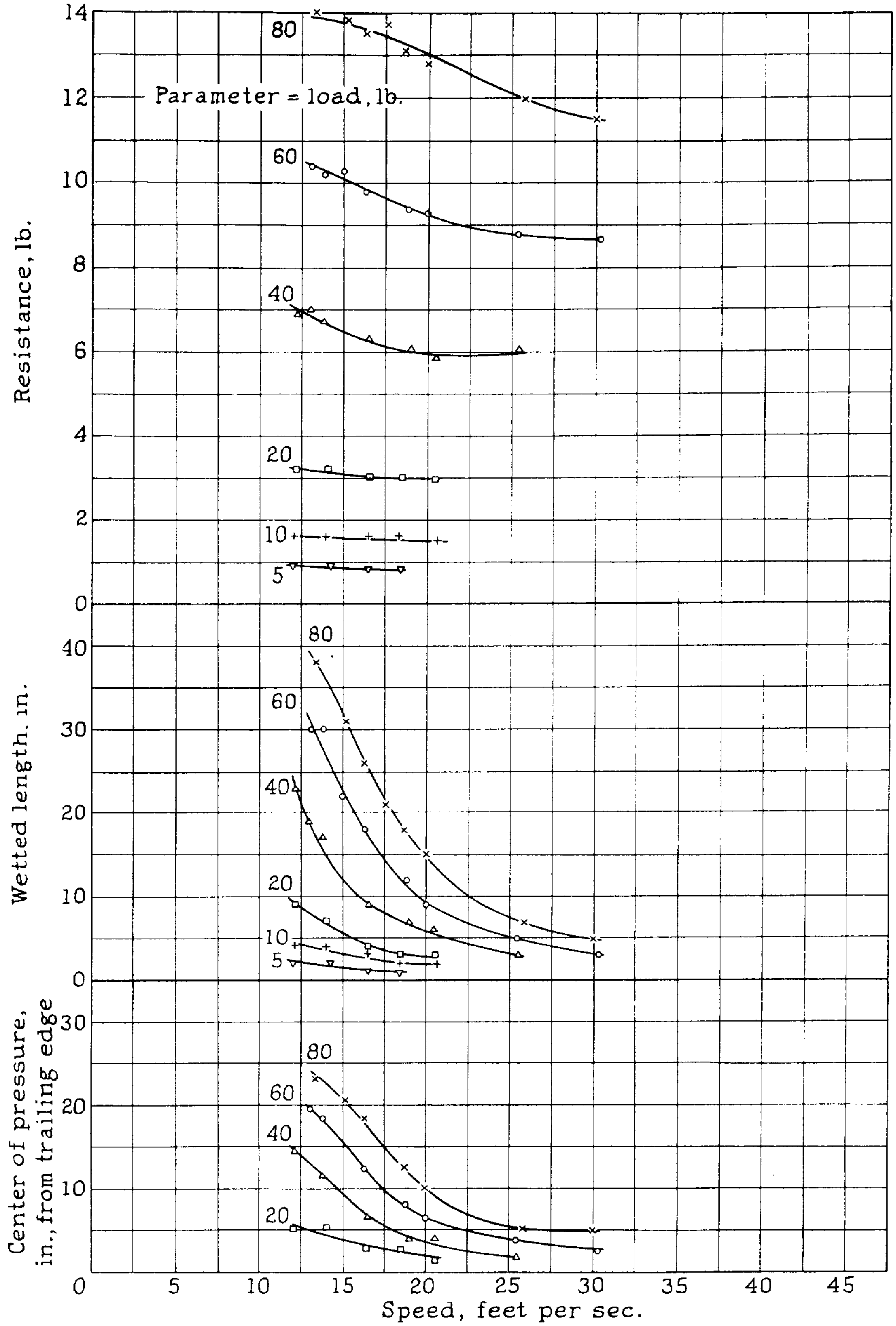

Figure 6. - Resistance, wetted length and center of pressure. Model 27, 0° dead rise, τ = 8°.

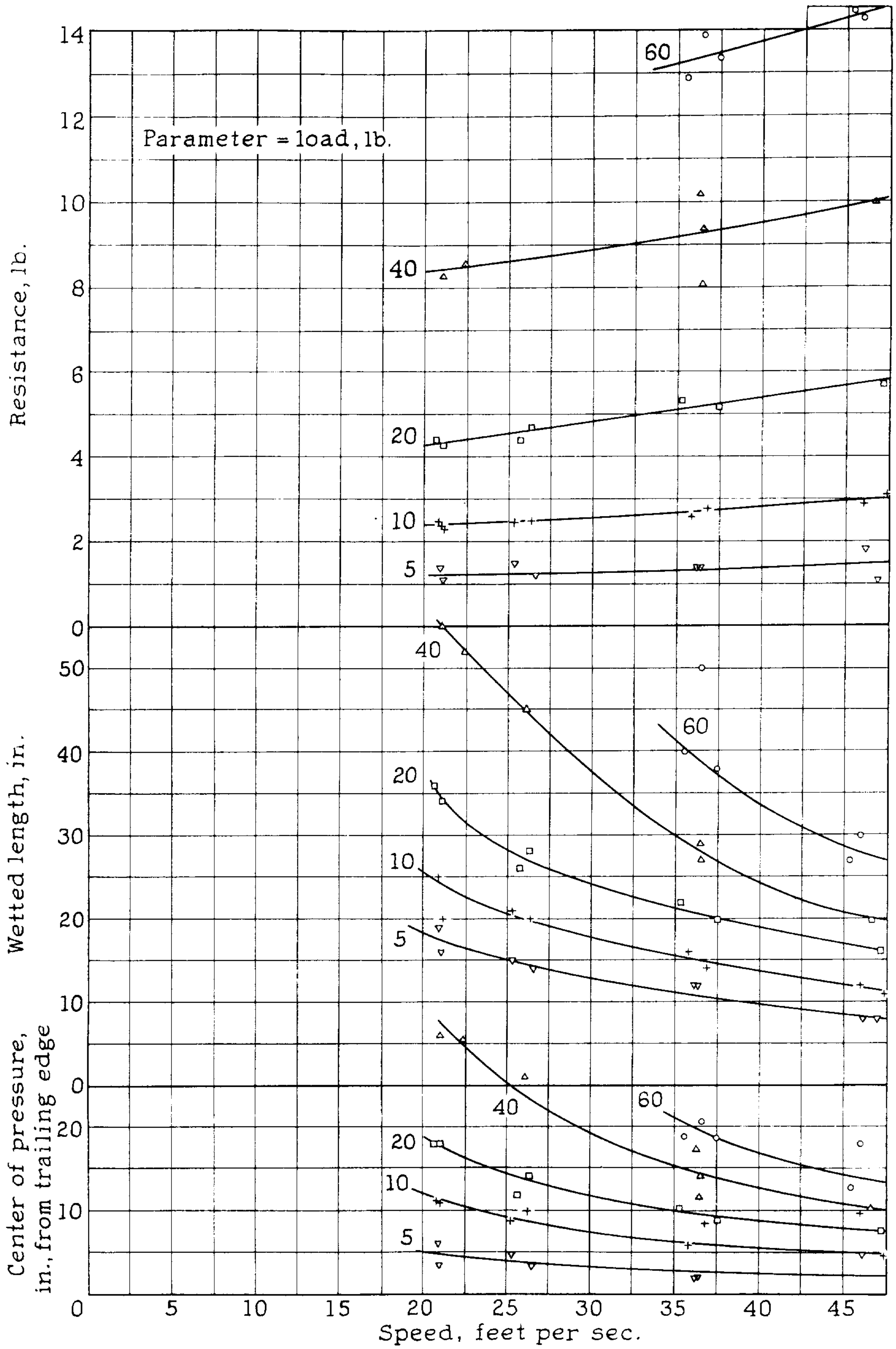

Figure 7. - Resistance, wetted length and center of pressure. Model 28, 10° dead rise. τ = 2°

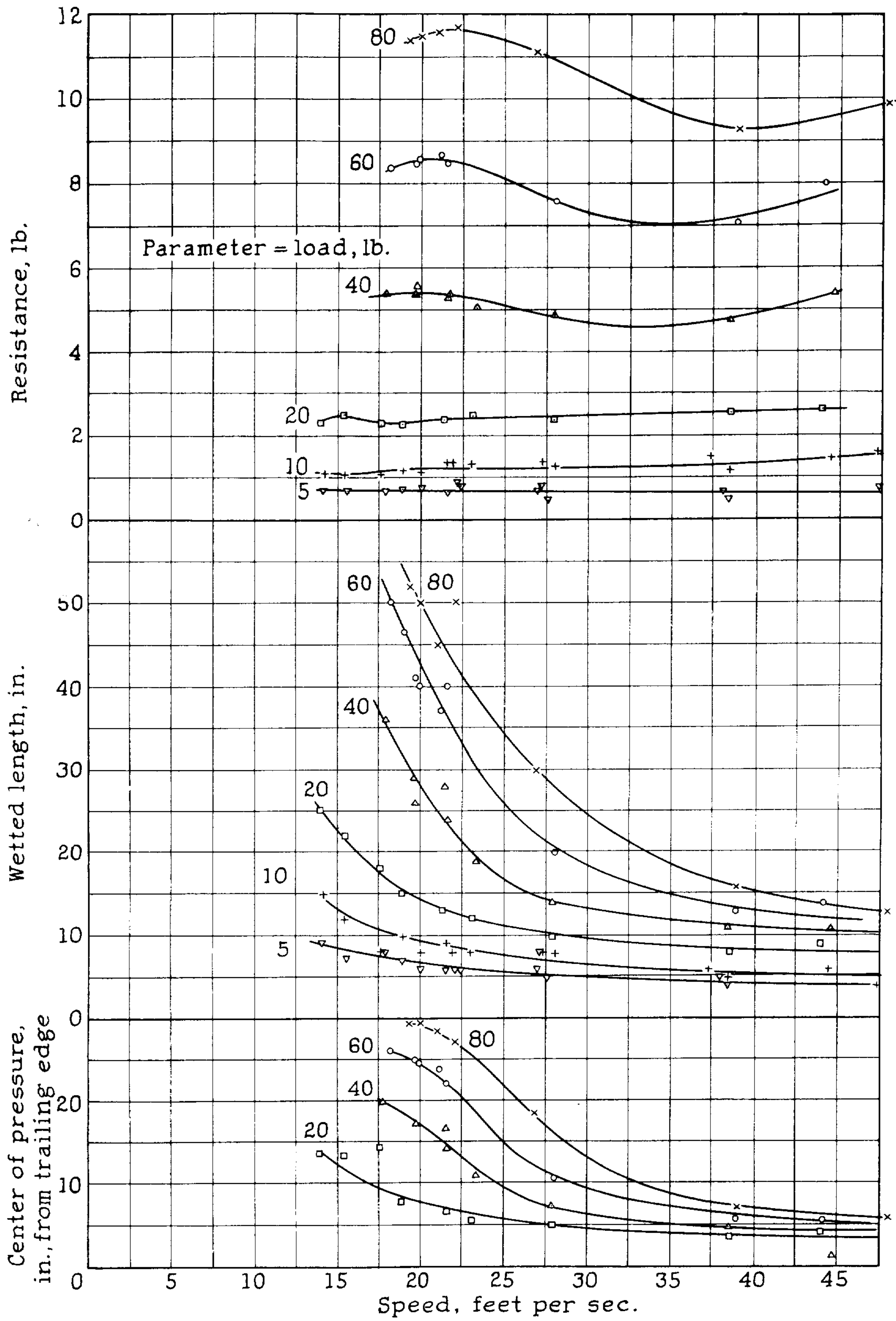

Figure 8. - Resistance, wetted length and center of pressure. Model 28, 10° dead rise. τ = 4°

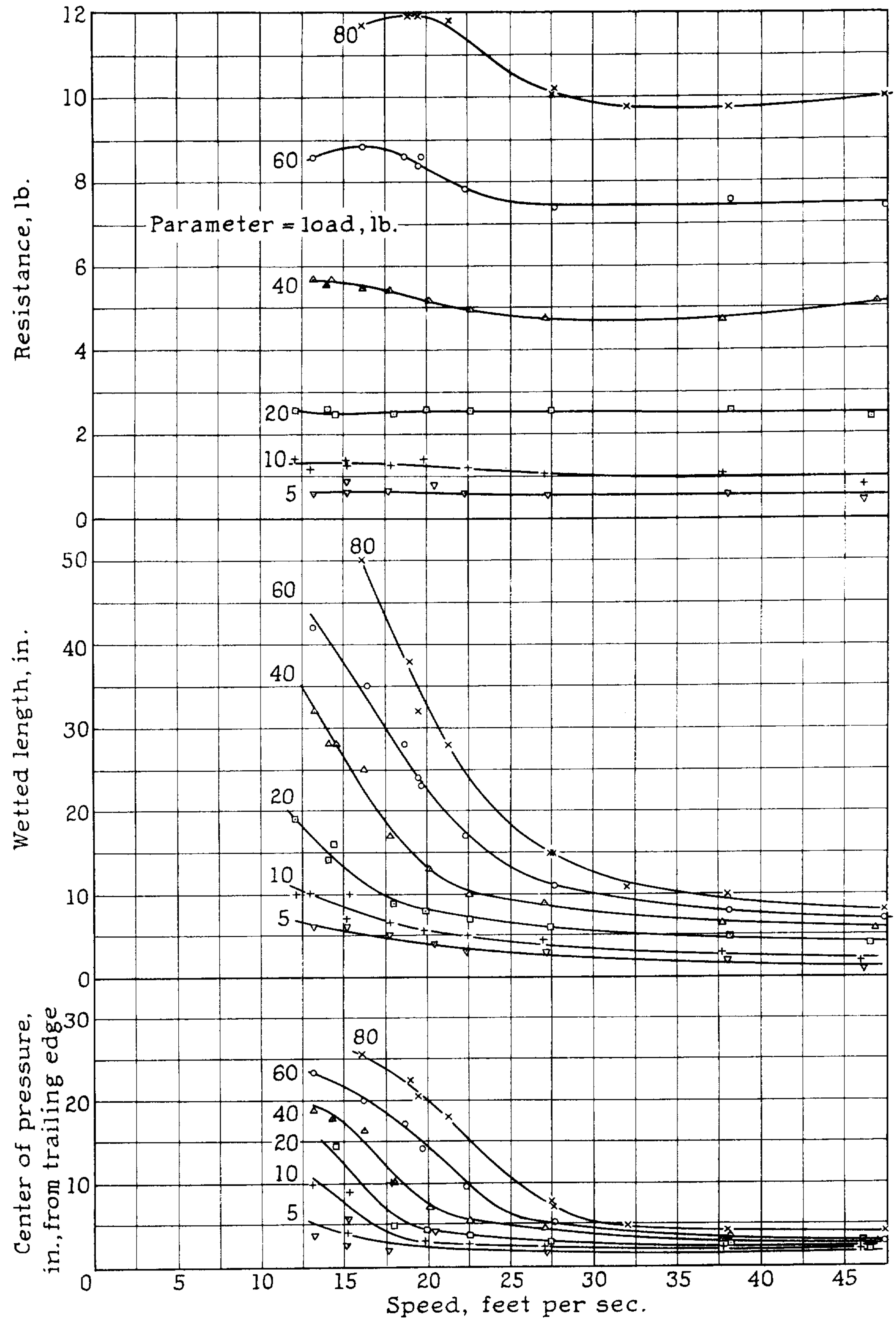

Figure 9. - Resistance, wetted length and center of pressure. Model 28, 10° dead rise. $\tau = 6°$.

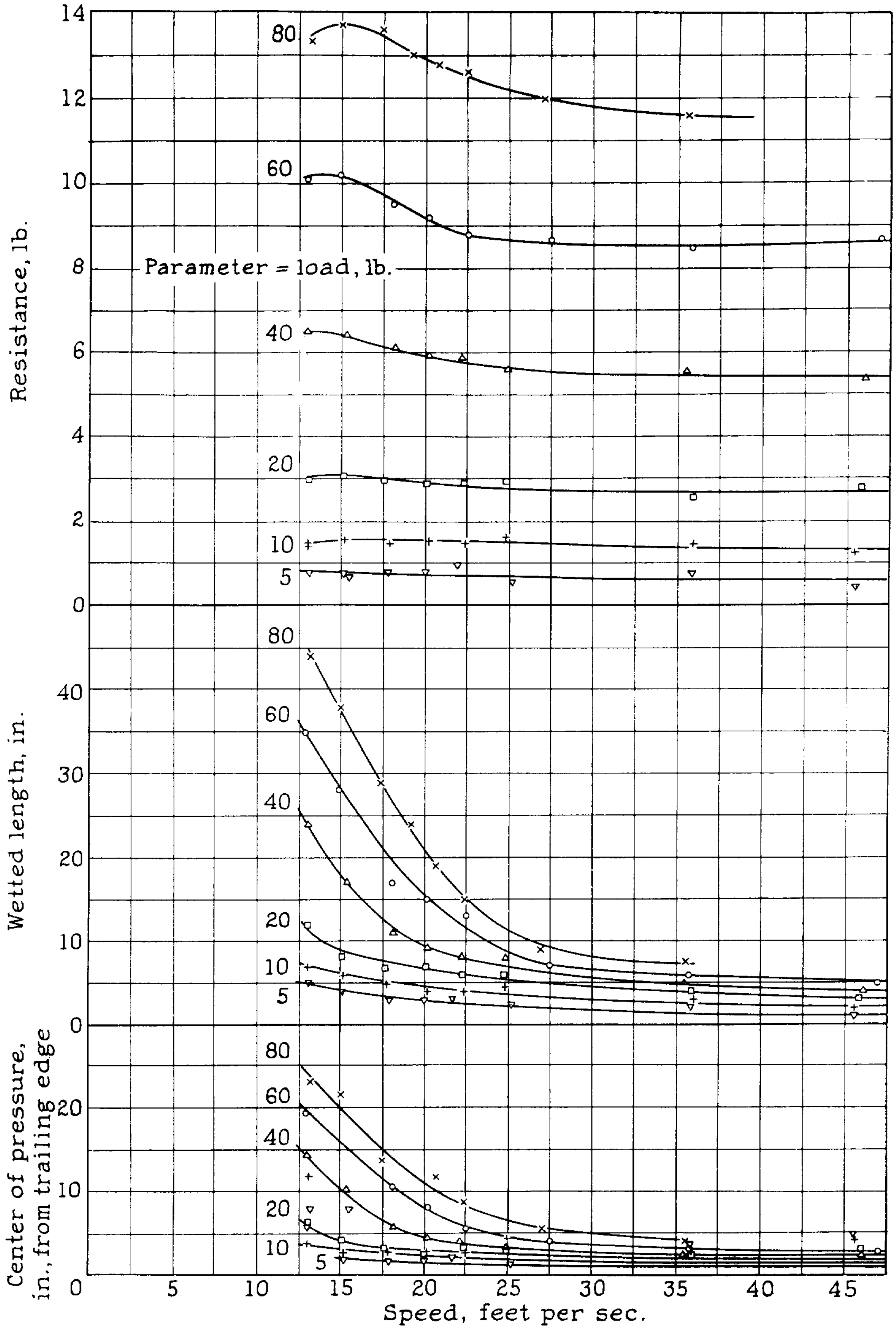

Figure 10. - Resistance, wetted length and center of pressure. Model 28, 10° dead rise. $\tau = 8°$.

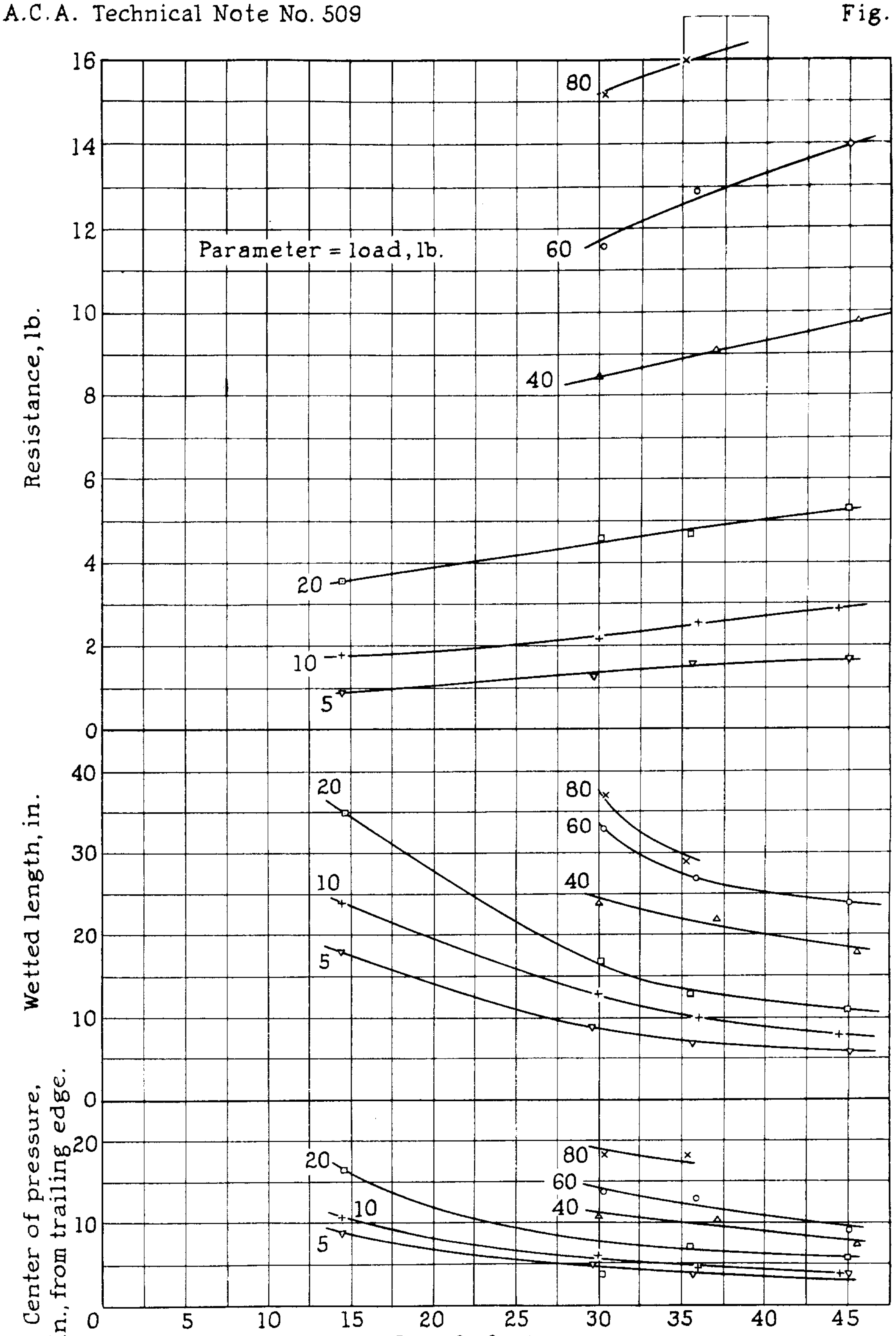

Figure 11. - Resistance, wetted length and center of pressure. Model 29, 20° dead rise. τ = 4°.

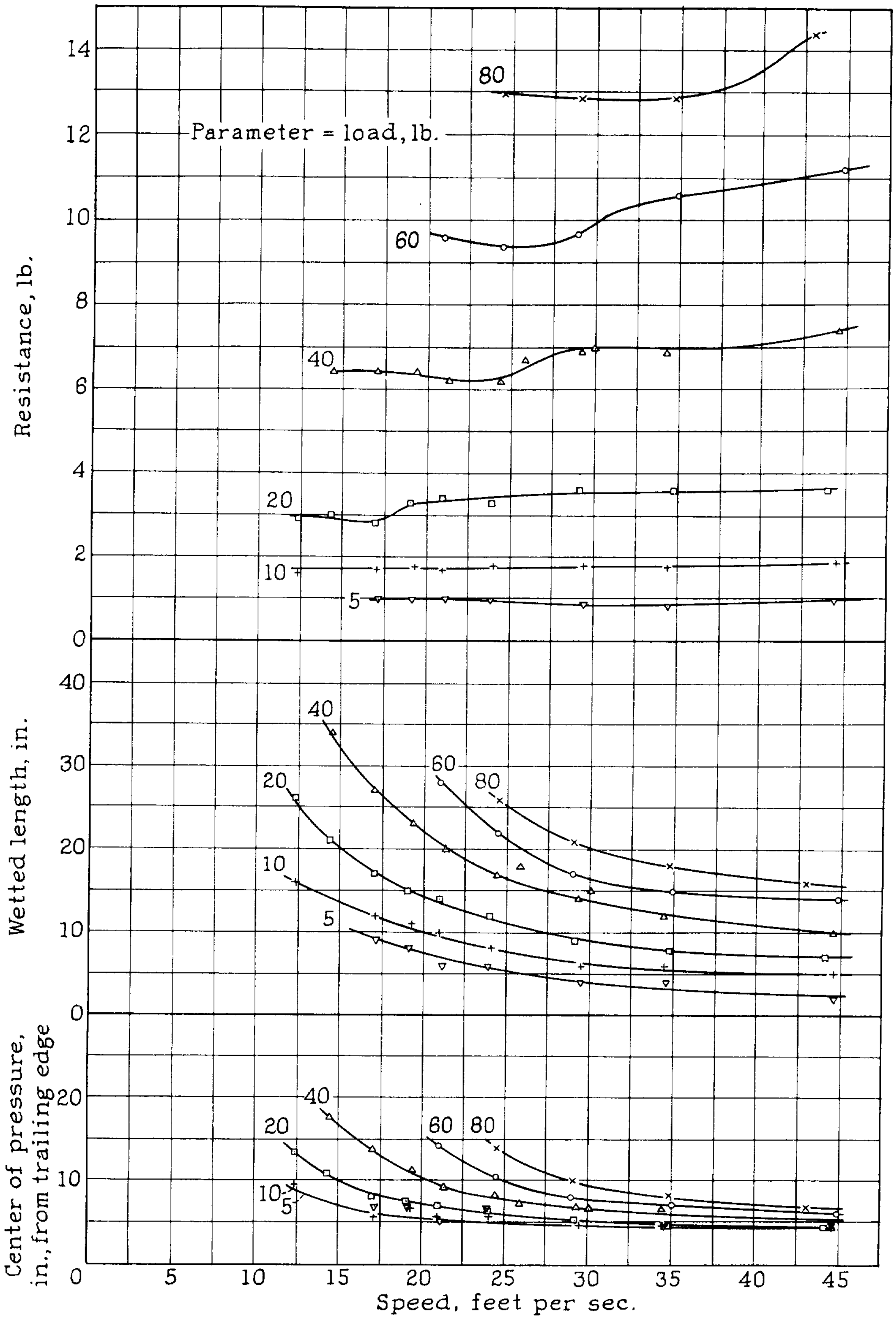

Figure 12.- Resistance, wetted length and center of pressure. Model 29, 20° dead rise. τ = 6°

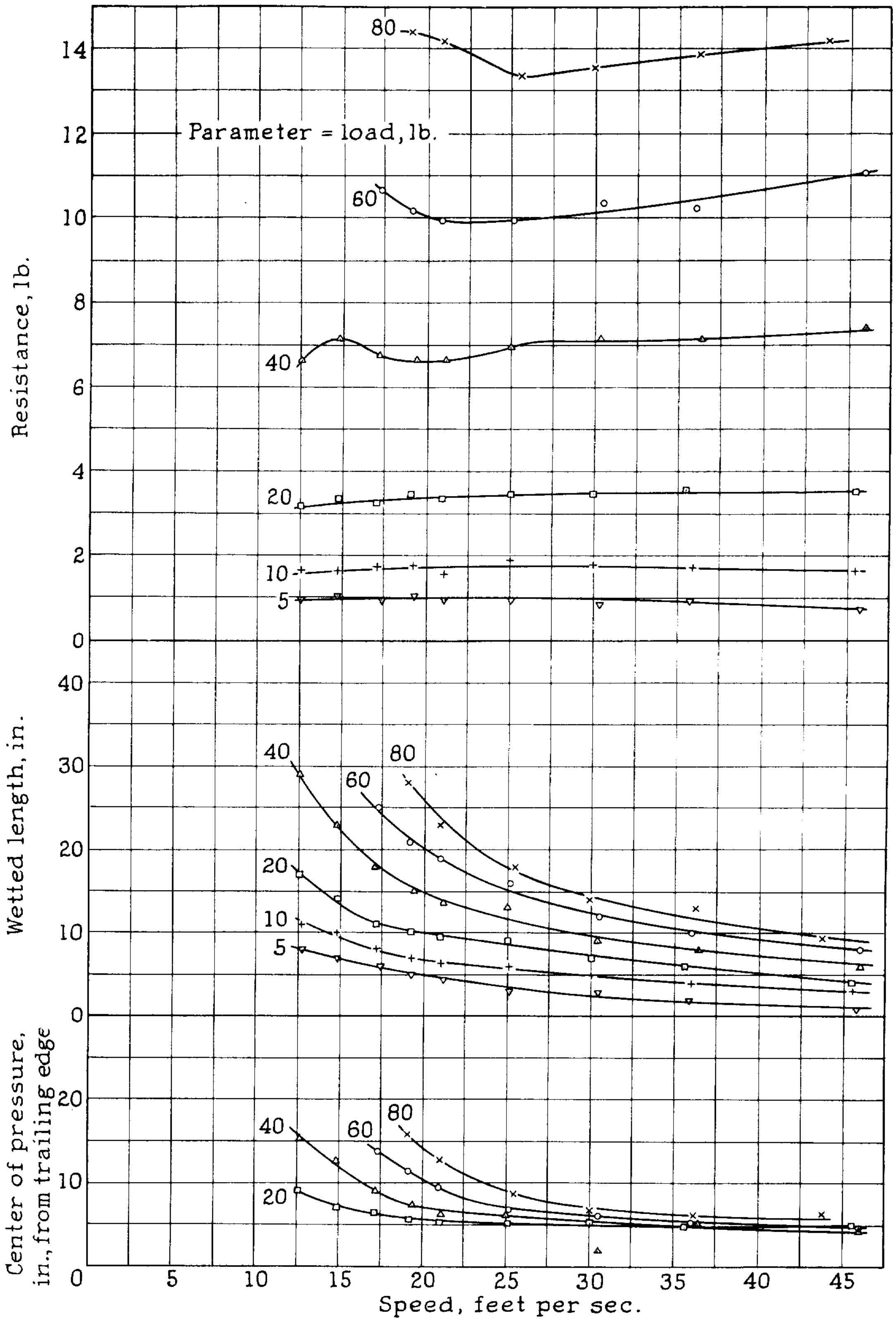

Figure 13. - Resistance, wetted length and center of pressure. Model 29, 20° dead rise. $\tau = 8°$.

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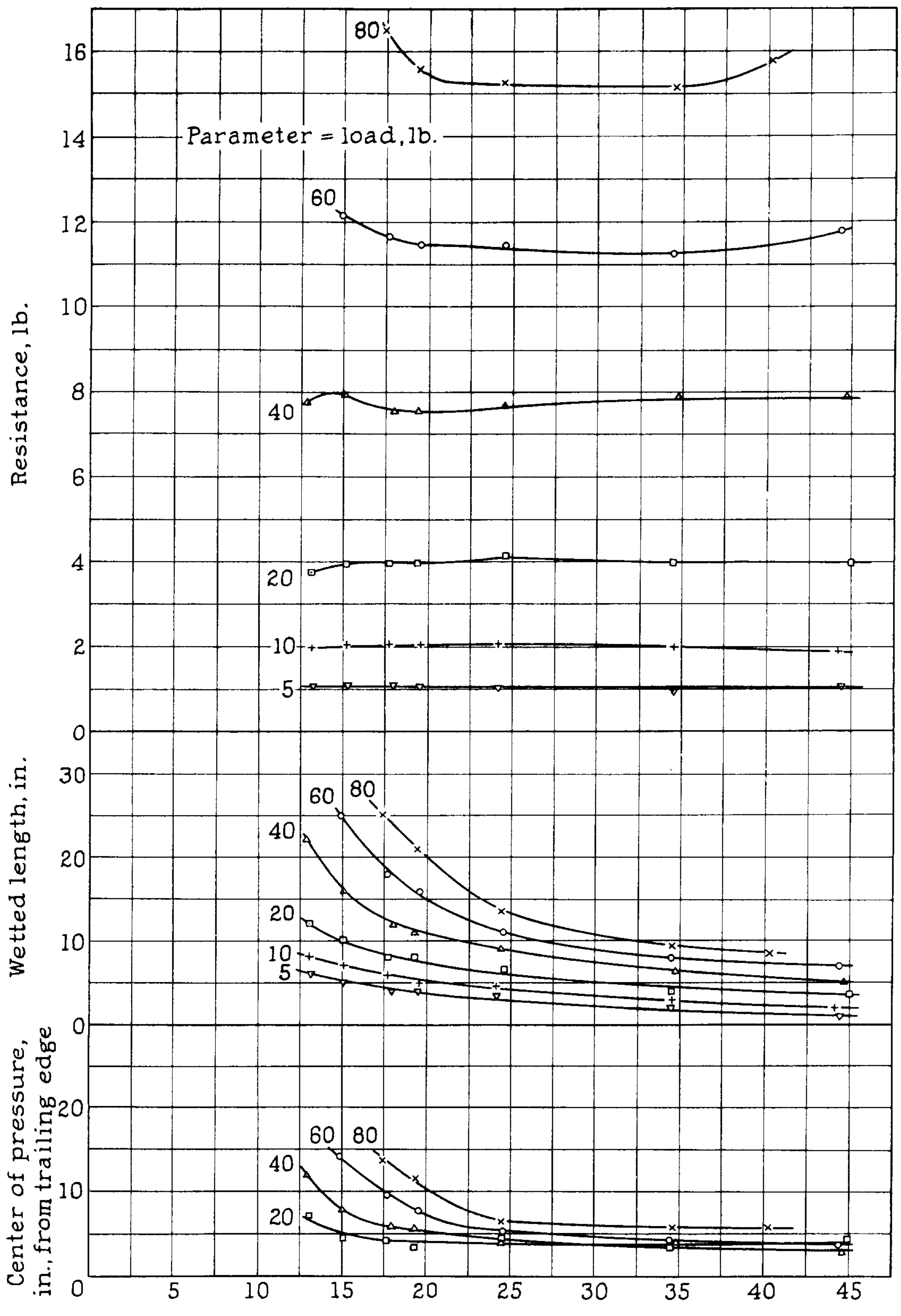

Figure 14. - Resistance, wetted length and center of pressure. Model 29, 20° dead rise. $\tau = 10°$

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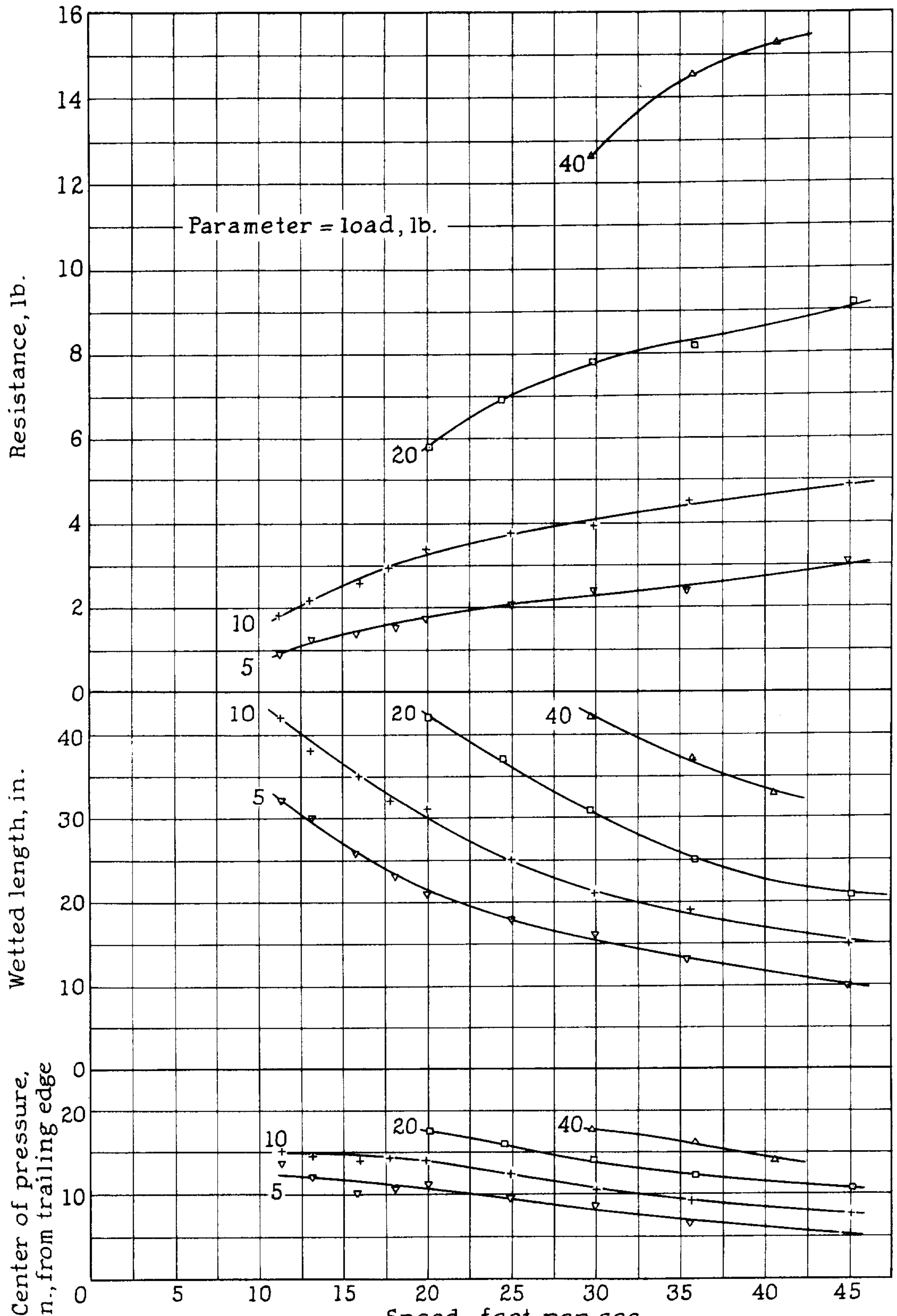

Figure 15. - Resistance, wetted length and center of pressure. Model 30, 30° dead rise, $\tau = 4°$.

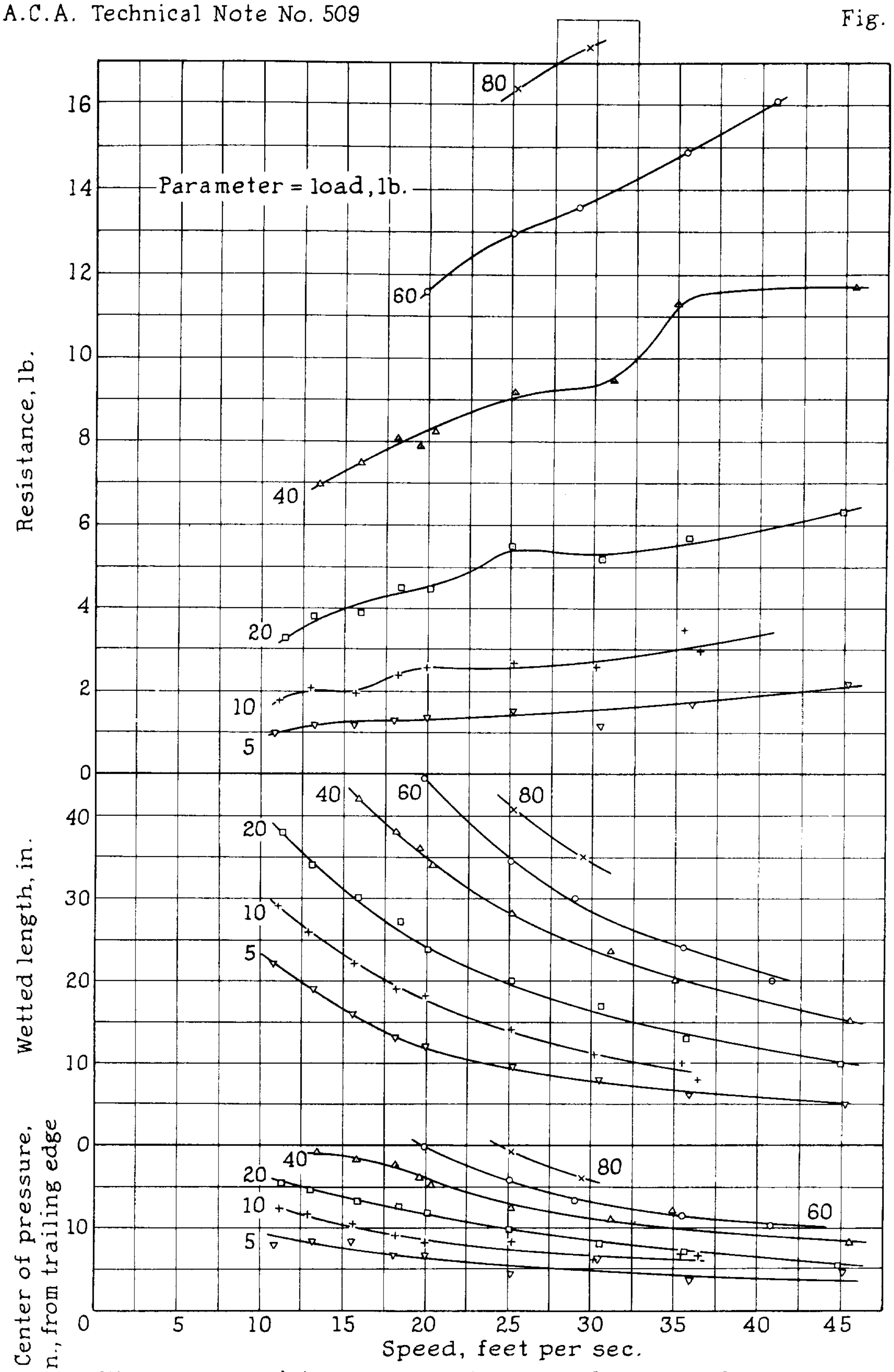

Figure 16. - Resistance, wetted length and center of pressure. Model 30, 30° dead rise. τ = 6°.

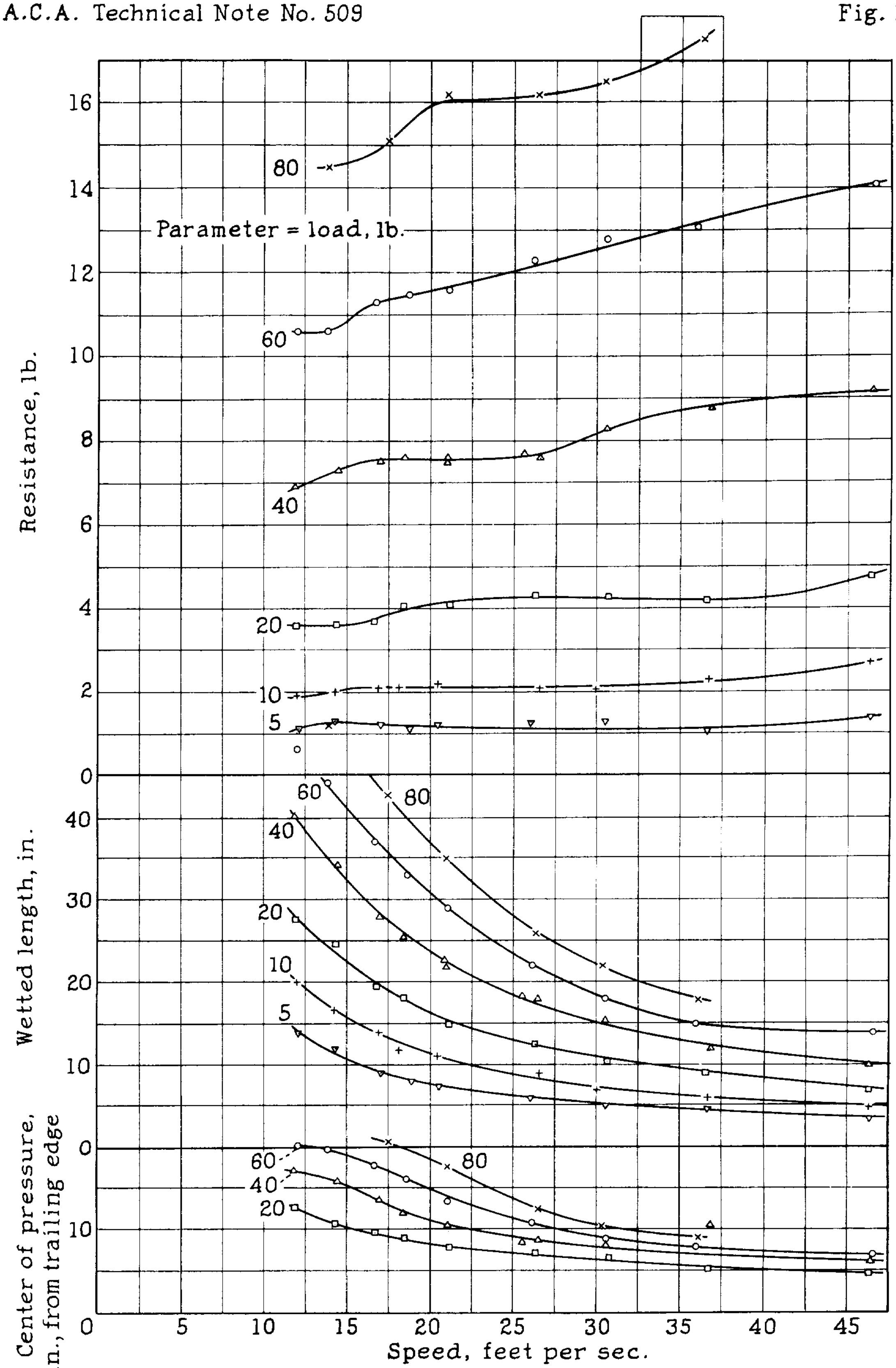

Figure 17. - Resistance, wetted length and center of pressure. Model 30, 30° dead rise. τ = 8°

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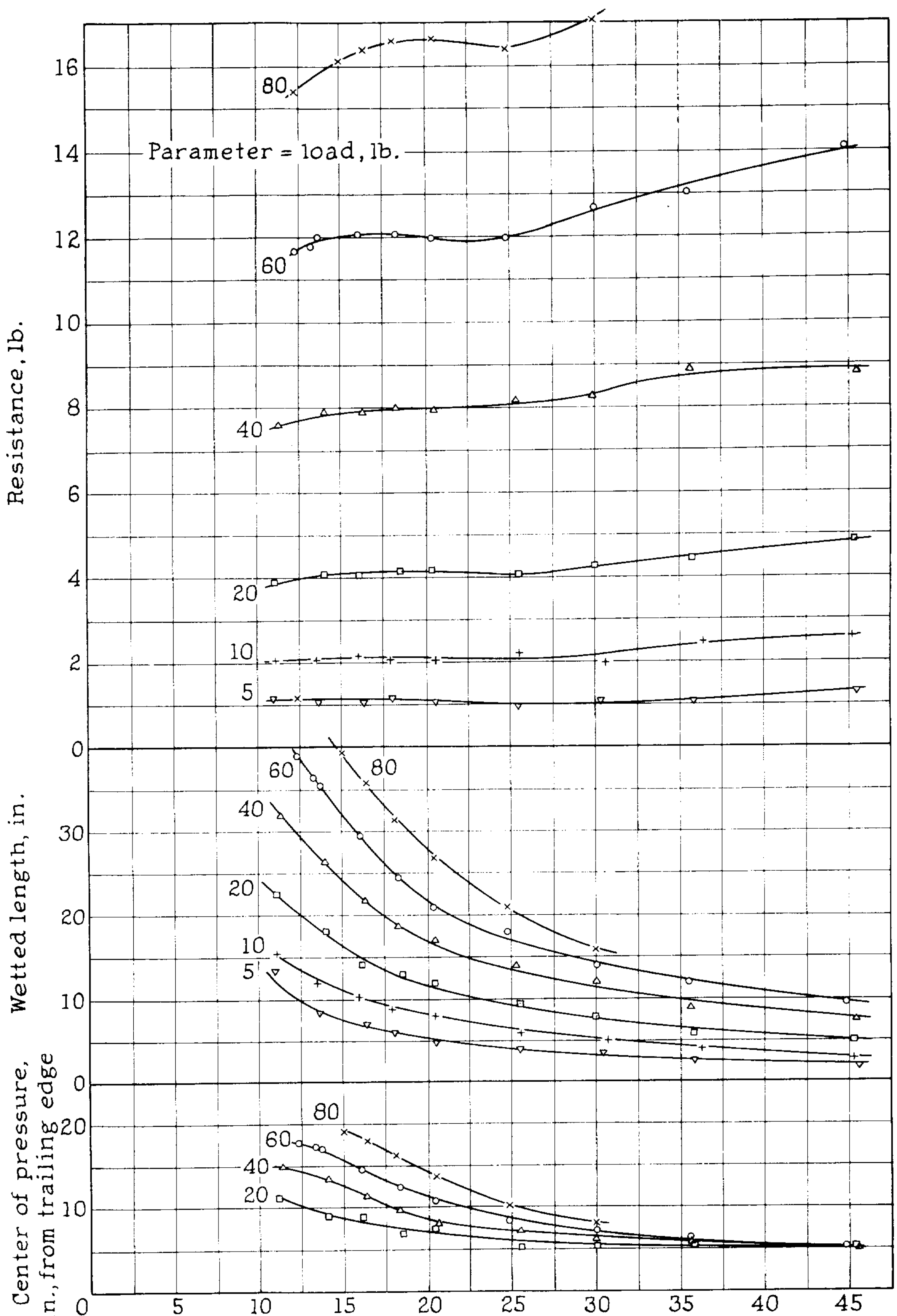

Figure 18. - Resistance, wetted length and center of pressure. Model 30, 30° dead rise, $\tau = 10°$

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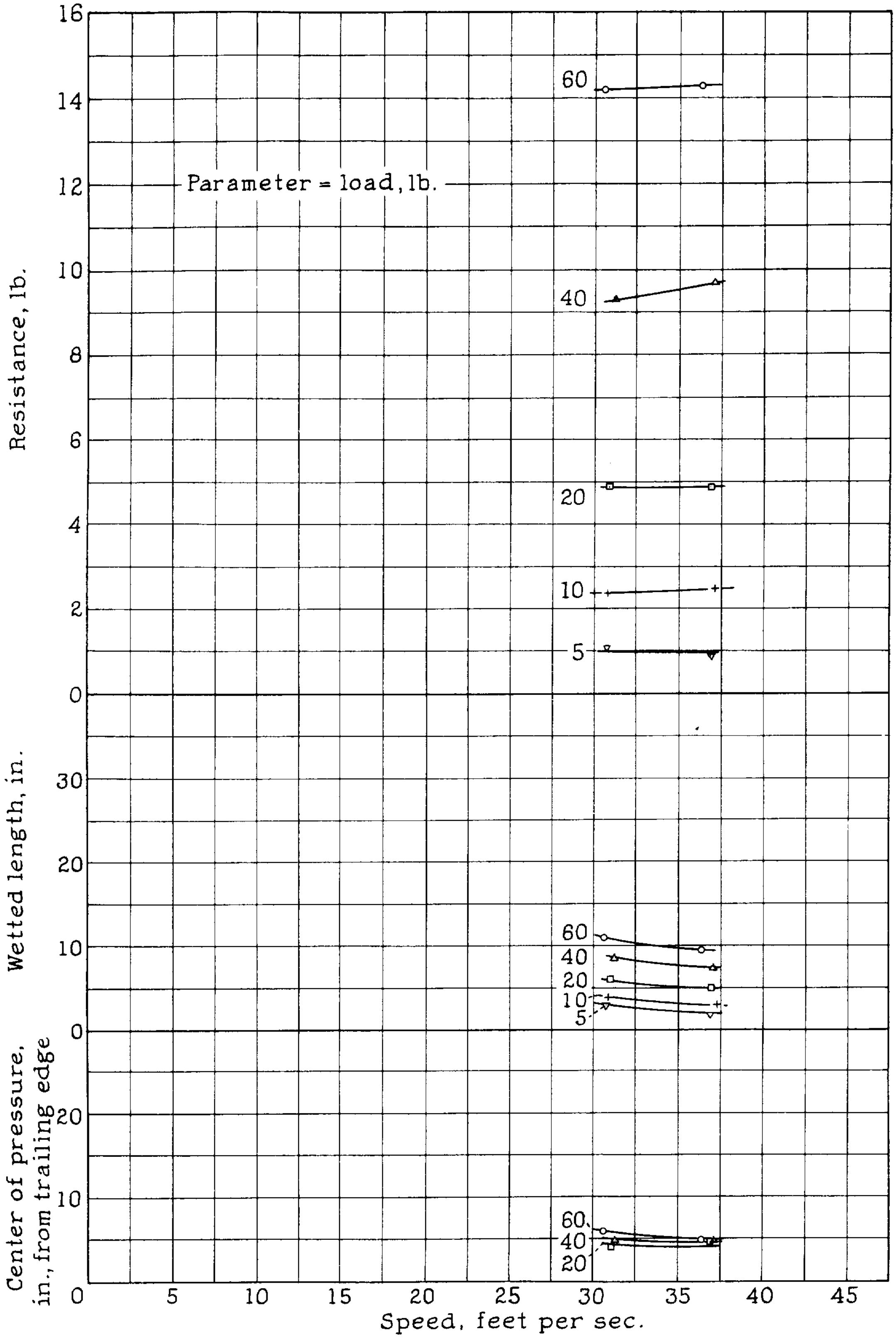

Figure 19. - Resistance, wetted length and center of pressure. Model 30, 30° dead rise. τ = 12°.

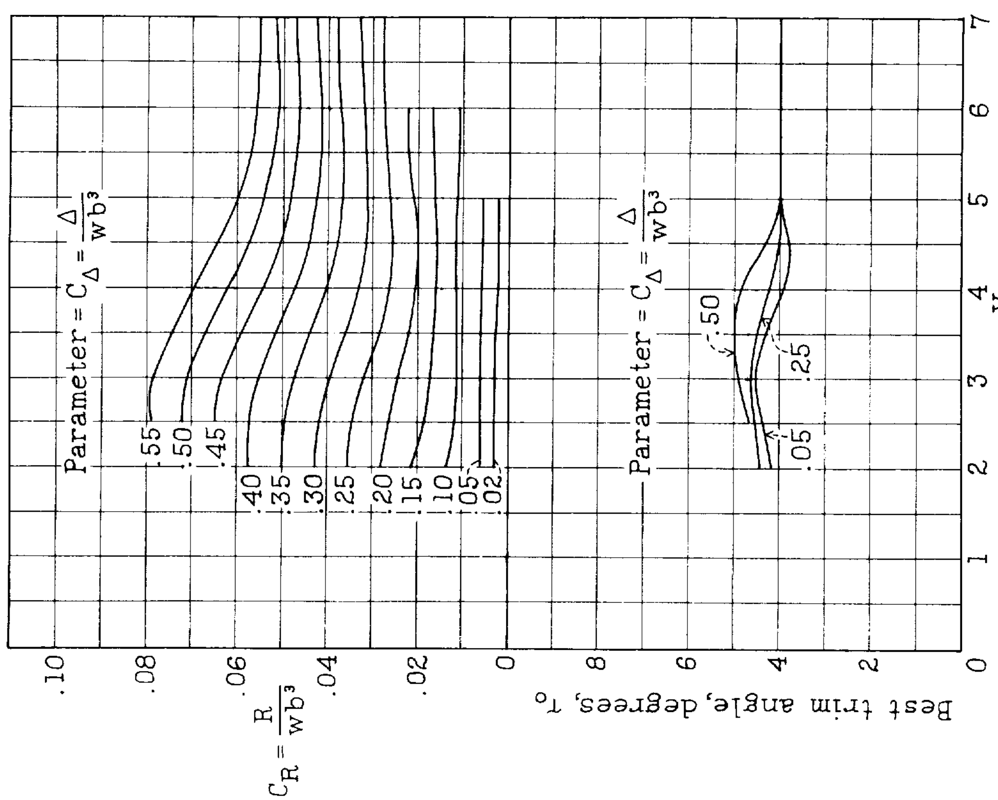

Figure 20. - Variation of C_R at best trim angle and τ_o with C_V. Model 27, 0° dead rise.

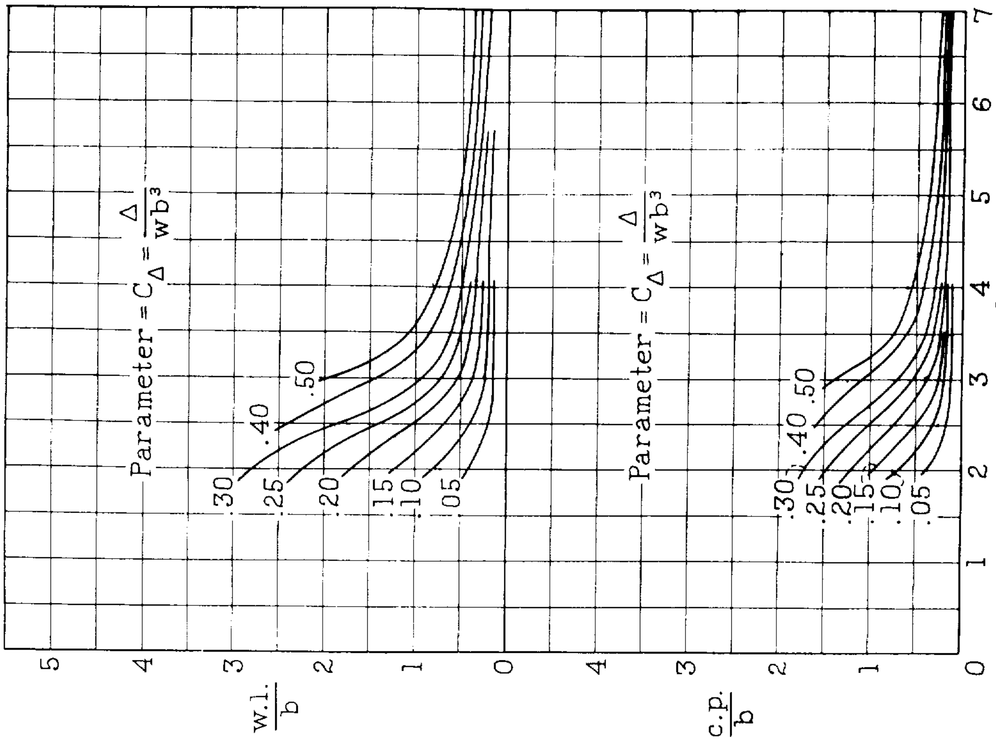

Figure 21. - Variation of w.l./b and c.p./b at best trim angle with C_V. Model 27, 0° dead rise.

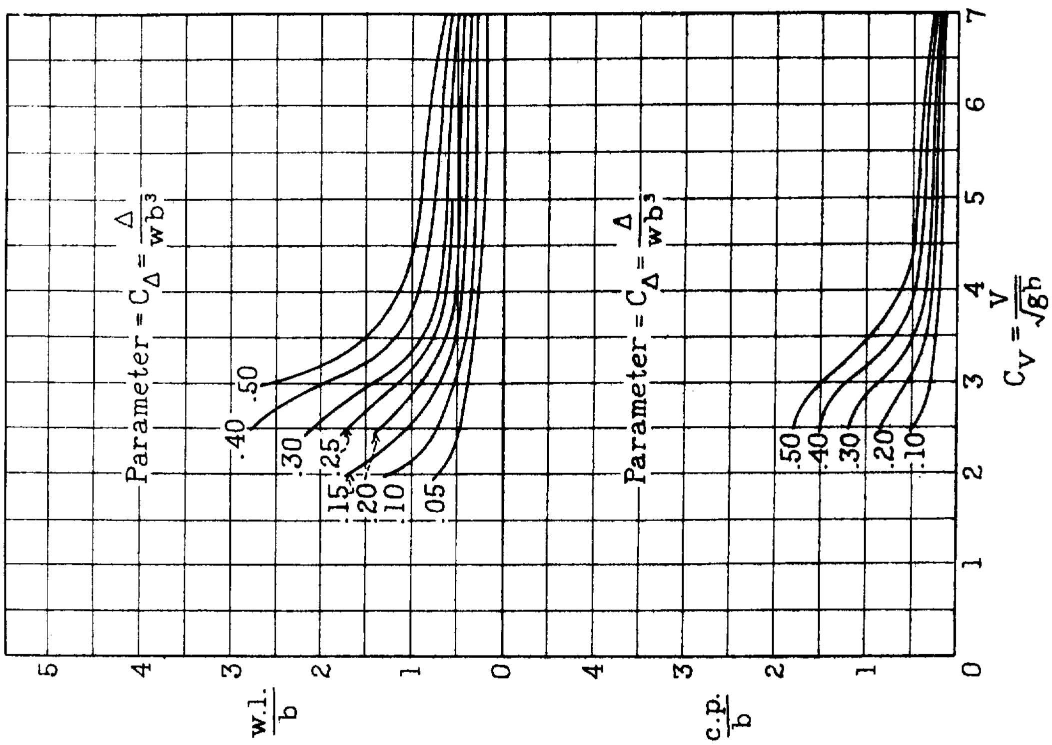

Figure 23. - Variation of w.l./b and c.p./b at best trim angle with C_V. Model 28, 10° dead rise.

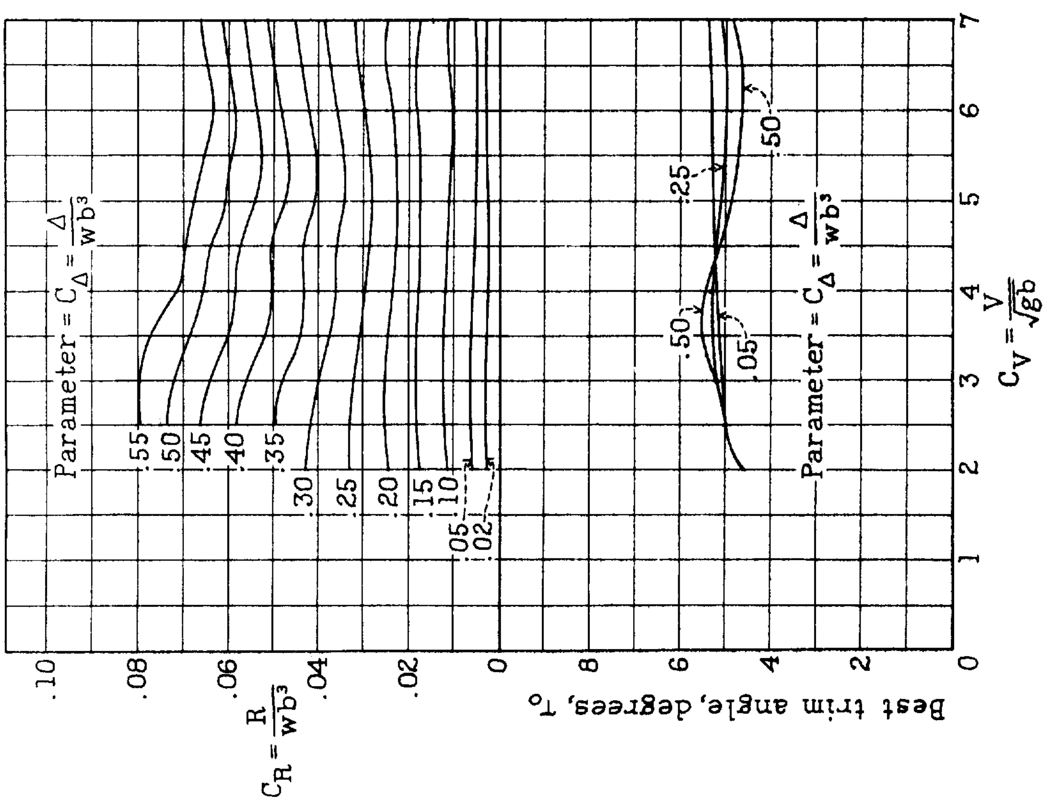

Figure 22. - Variation of C_R at best trim angle and τ_o with C_V. Model 28, 10° dead rise.

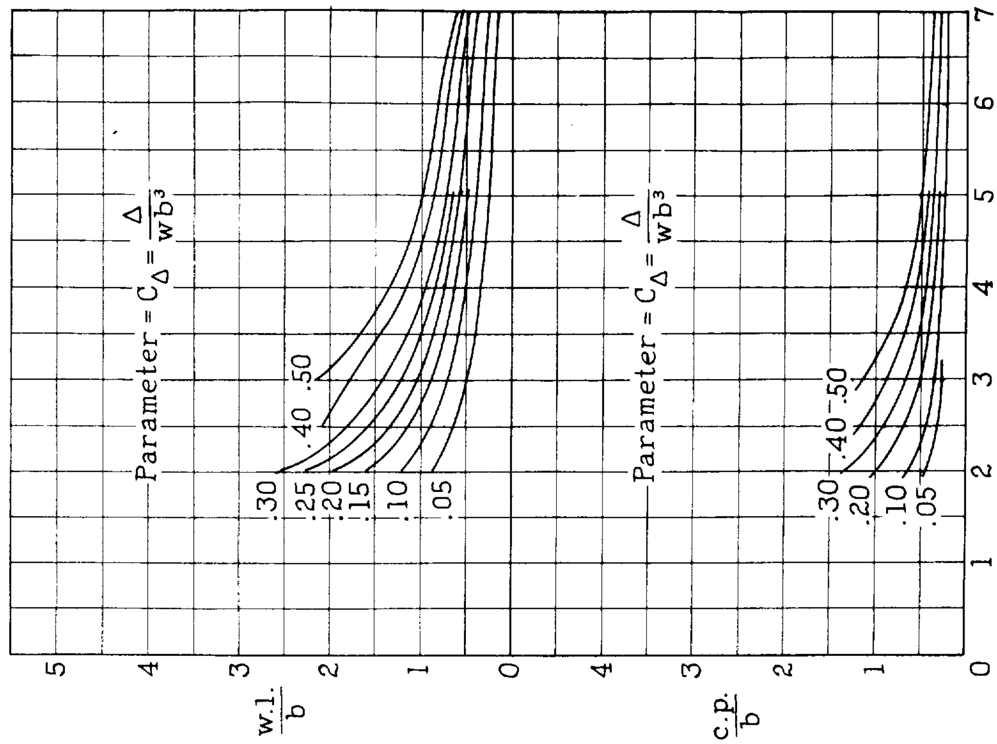

Figure 25. - Variation of w.l./b and c.p./b at best trim angle with C_V. Model 29, 20° dead rise.

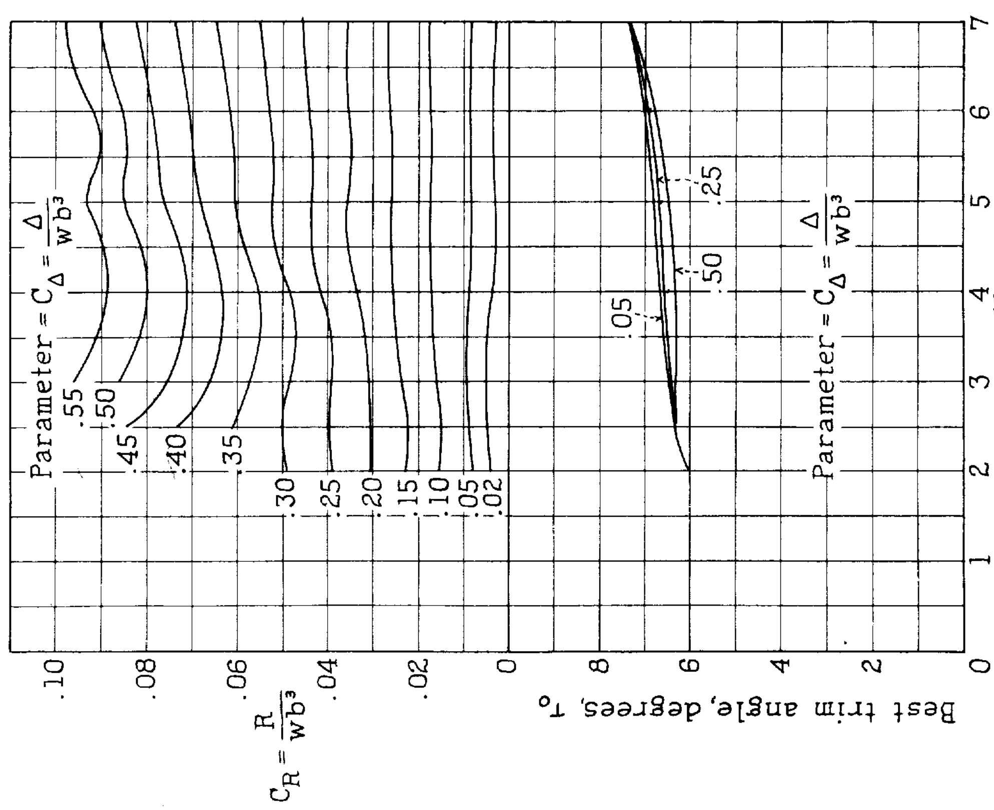

Figure 24. - Variation of C_R at best trim angle and τ_o with C_V. Model 29, 20° dead rise.

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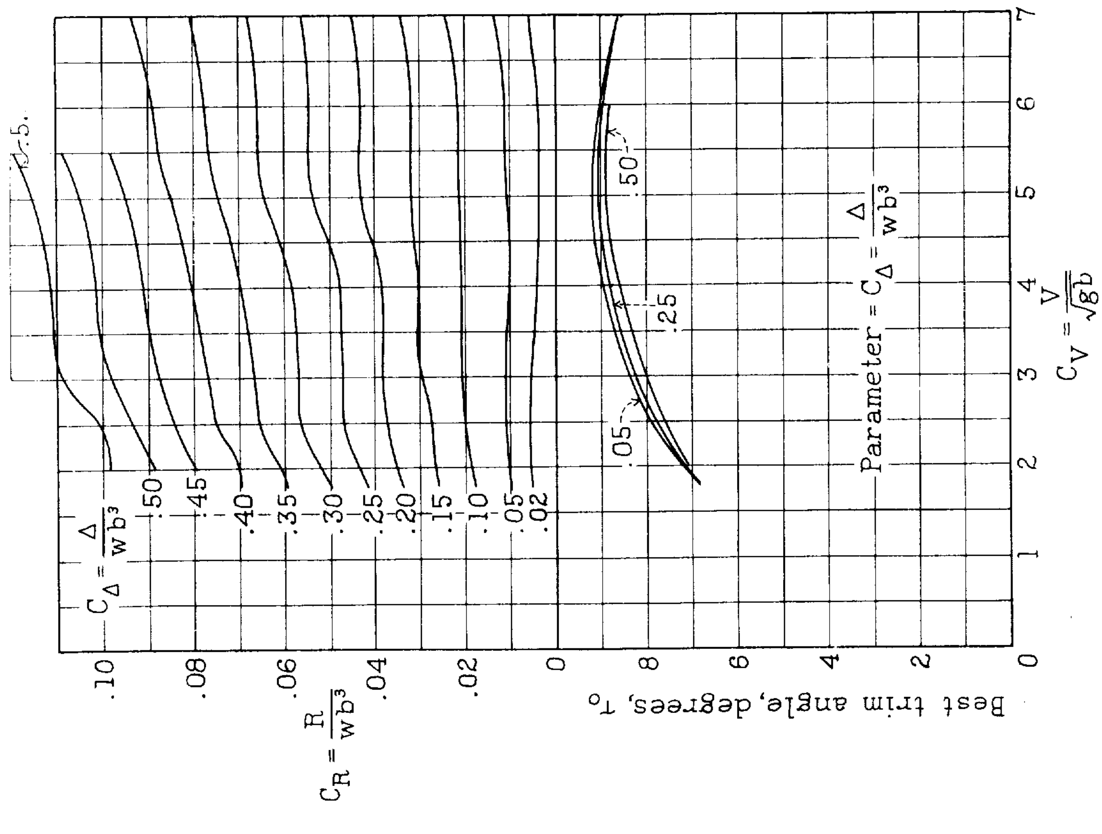

Figure 26. - Variation of C_R at best trim angle and τ_o with C_V. Model 30, 30° dead rise.

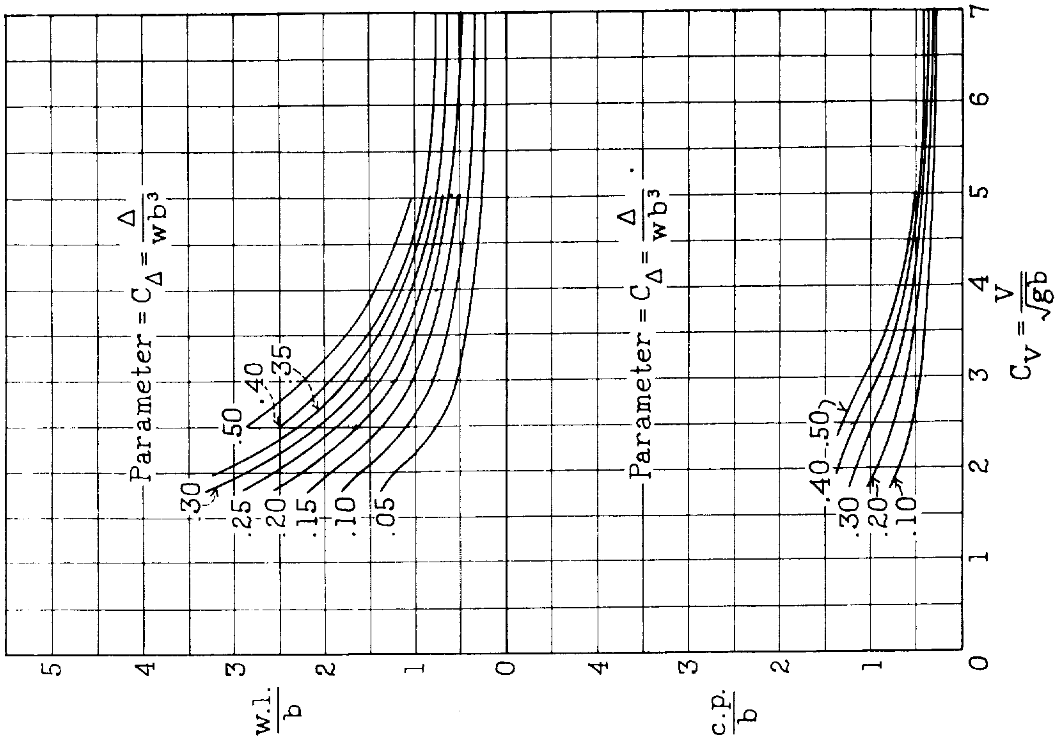

Figure 27. - Variation of w.l./b and c.p./b at best trim angle with C_V. Model 30, 30° dead rise.

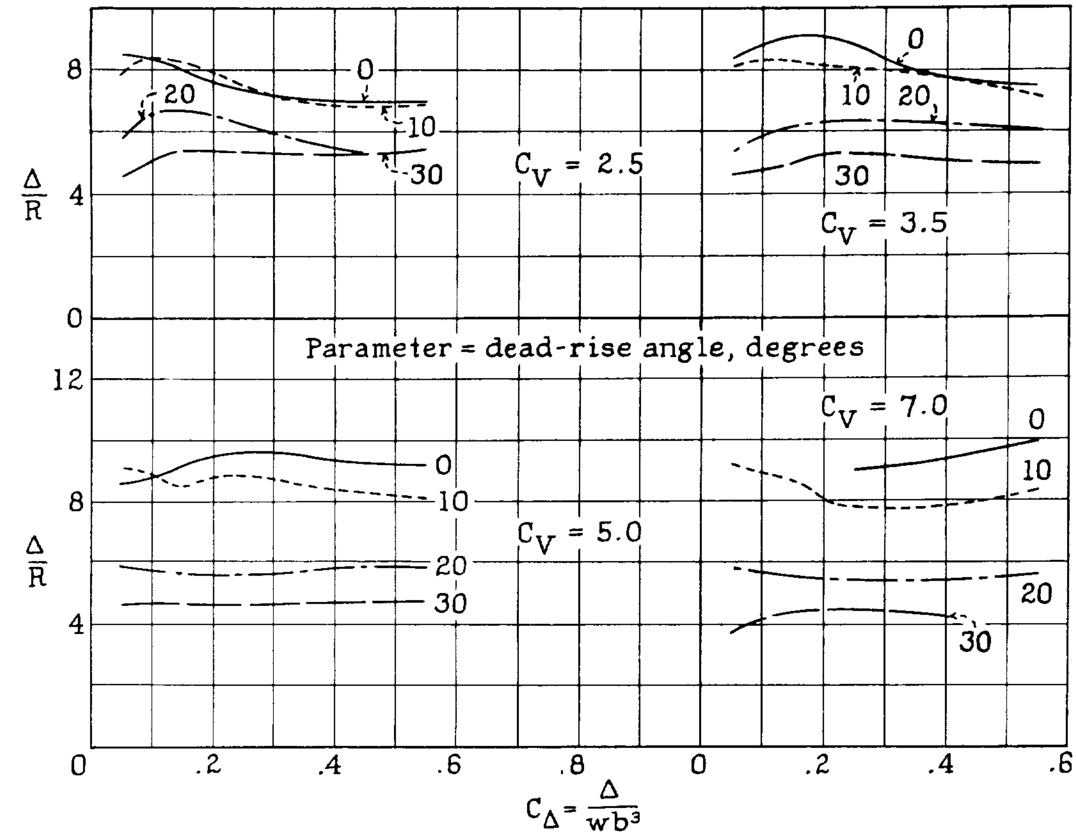

Figure 28.- Effect of dead rise on load/resistance ratio at best trim angles.

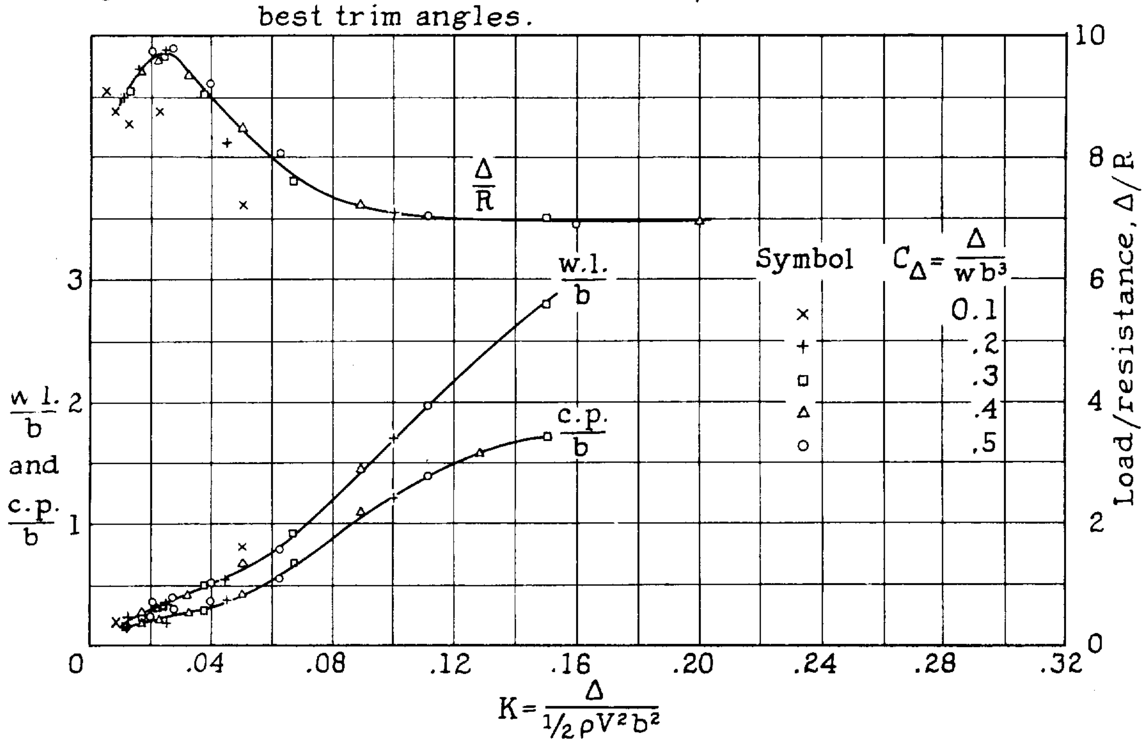

Figure 29.- Variation of Δ/R, w.l./b, and c.p./b at best trim angle with planing coefficient, K. Model 27, 0° dead rise.

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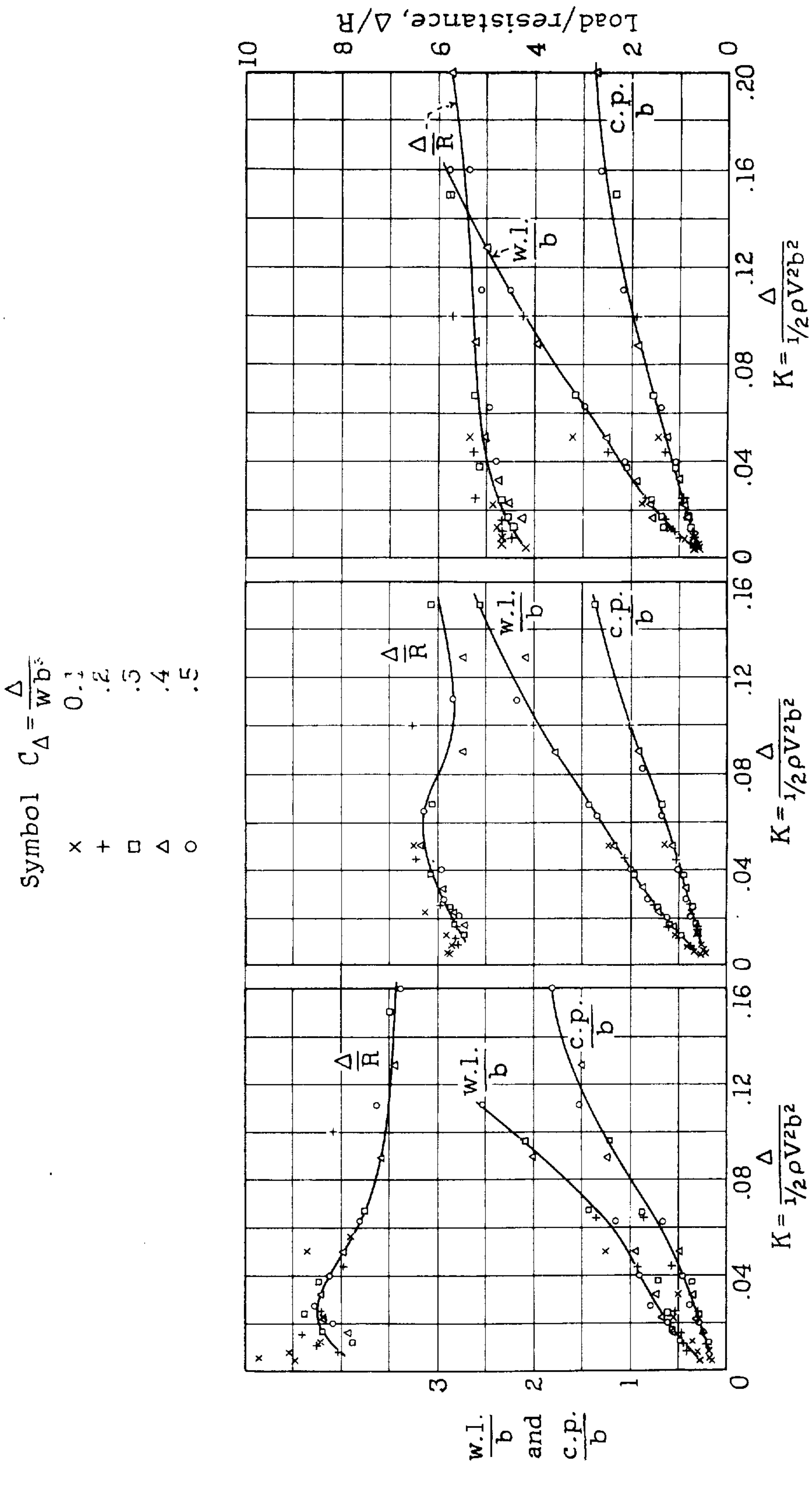

Figure 30.- Model 28, 10° dead rise.

Figure 31.- Model 29, 20° dead rise.

Figure 32.- Model 30, 30° dead rise.

Variation of Δ/R, w.l./b, and c.p./b at best trim angle with planing coefficient, K.

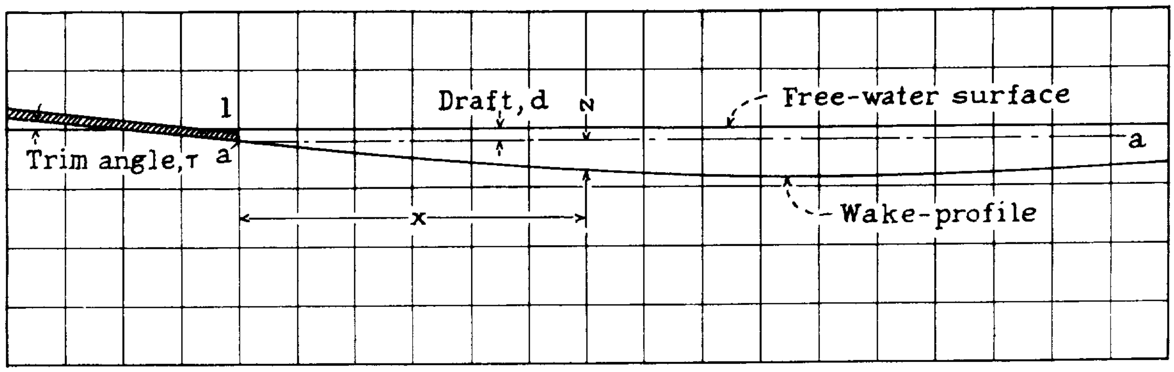

Figure 33.- Diagram of wake profile for flat surface.

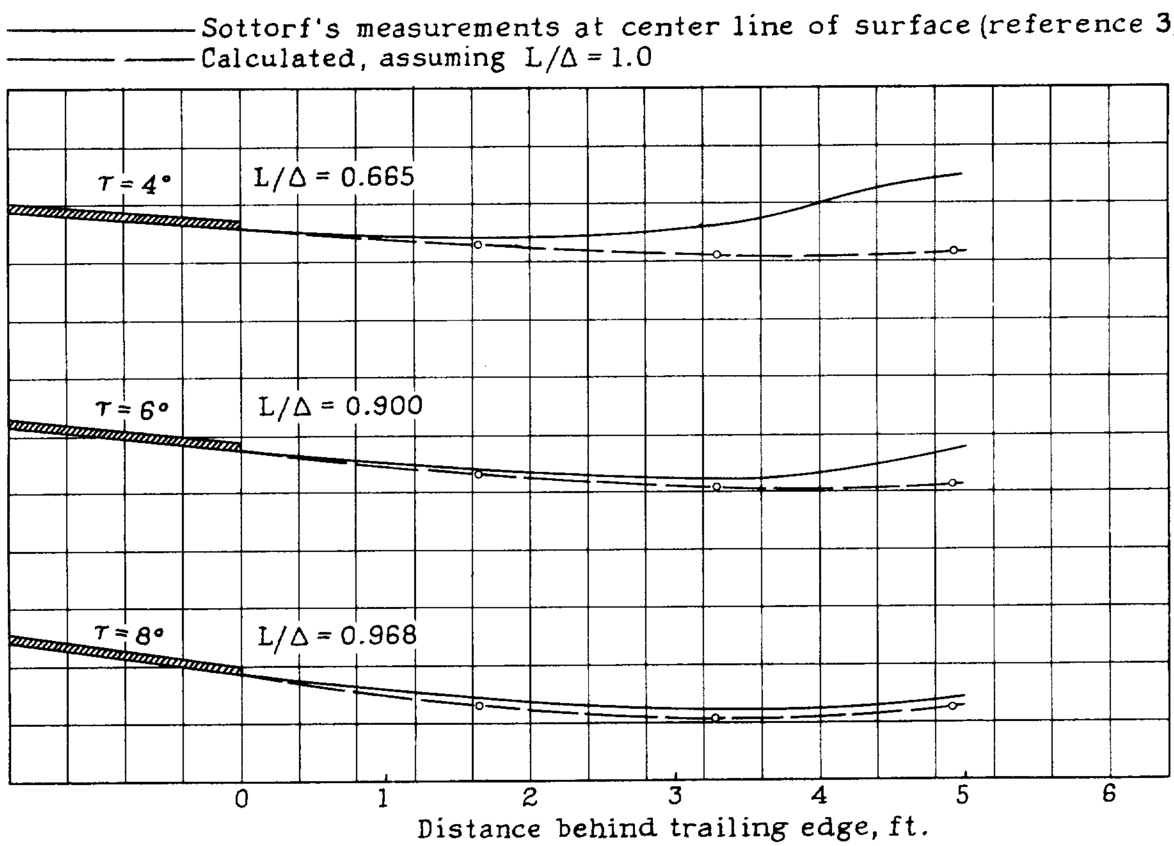

Figure 34.- Comparison of calculated and measured wake profiles behind flat surface. Beam = 0.985 ft., Δ = 39.7 lb., V = 19.7 feet per sec.

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